

Quantum aspects of topological photonics

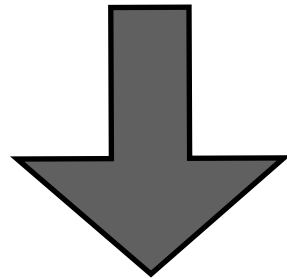
Mohammad Hafezi



Quantum Fluids of light and matter, Les Houches 2018



Topological states and beyond Photons vs. Electrons

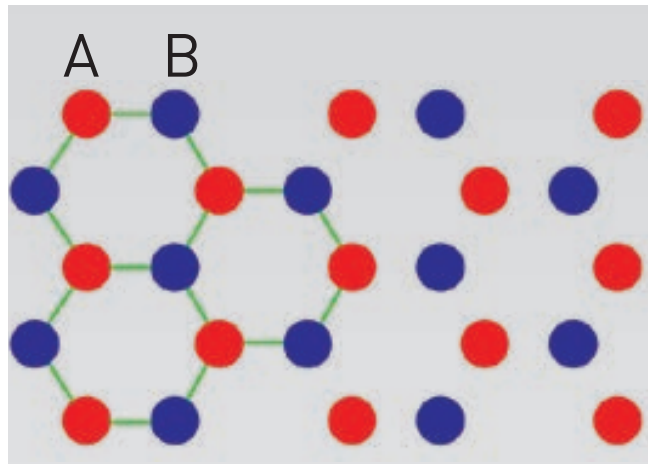


- Bosons vs. Fermion:
Preparation, measurement, order type
- Energy scale, room temperature
- Interaction (three-body, weak?)
- Length scale, Local addressability
(manipulation/detection)
- Anyons?

Outline of this talk

- Background
- Quantum direction of topological photonics:
 - Topologically robust generation of photons
 - Topological quantum optics interface
- Photons and electronic quantum Hall states

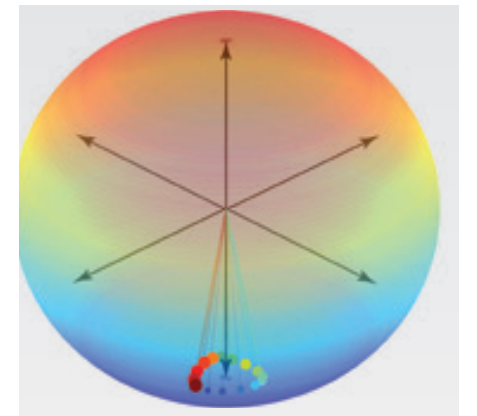
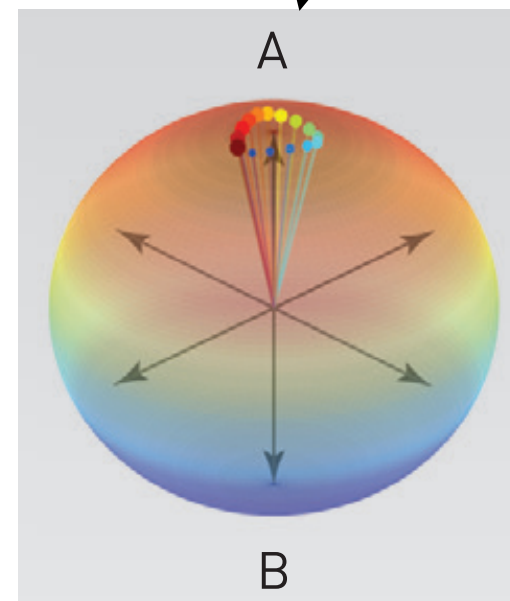
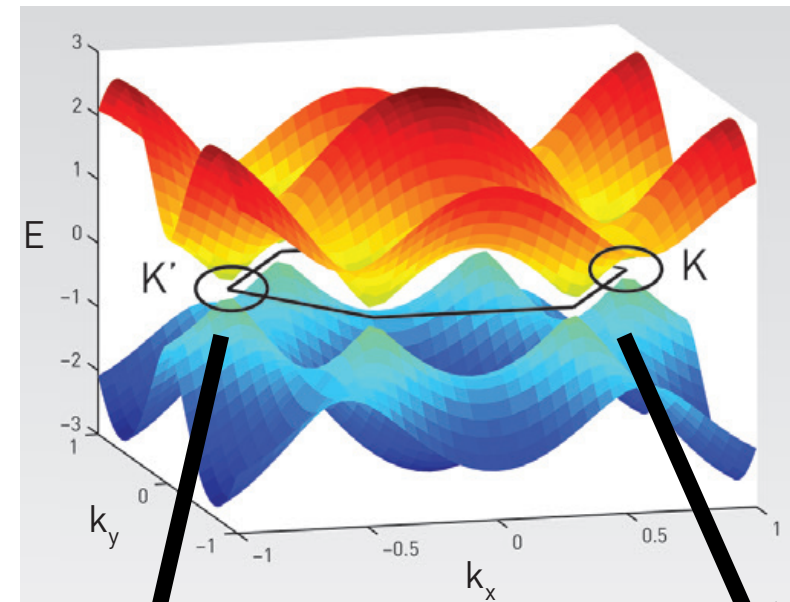
Photonic crystals
period dielectric structure



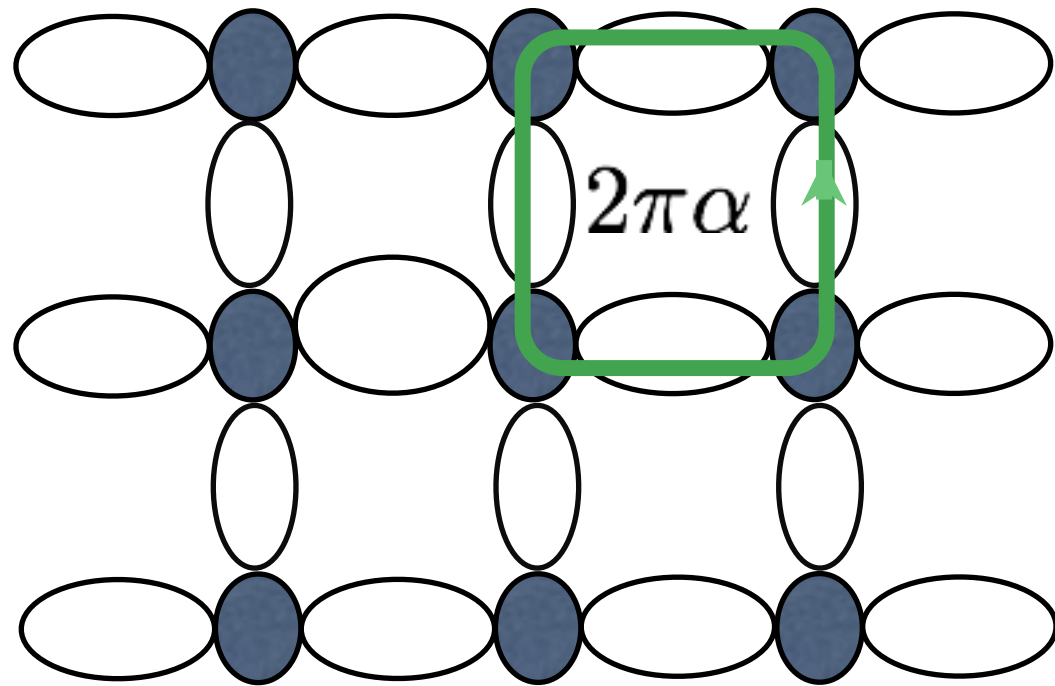
But there is something more!
global properties of the band
structure

Global features can not be
affected by local perturbation

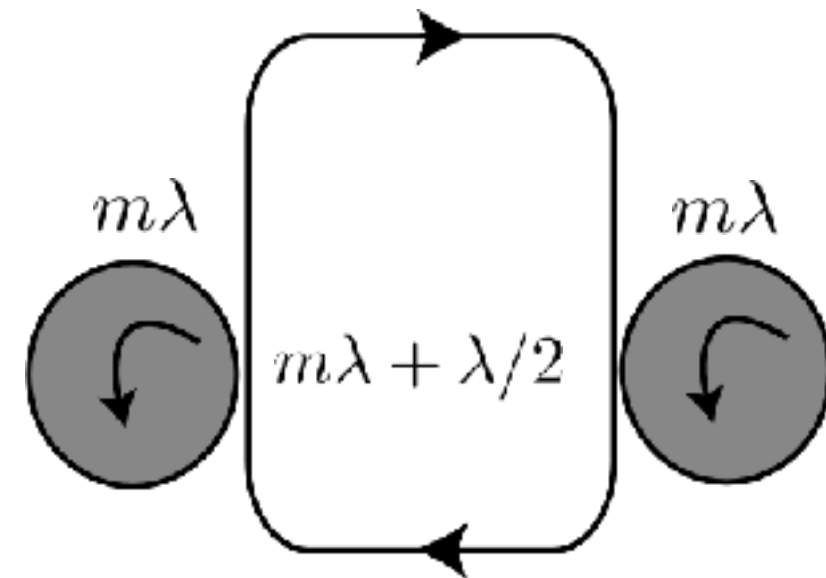
Band structure in Brillouin Zone
local properties: $E(k)$, band gap,
various bound and propagating features



Synthetic Magnetic Field

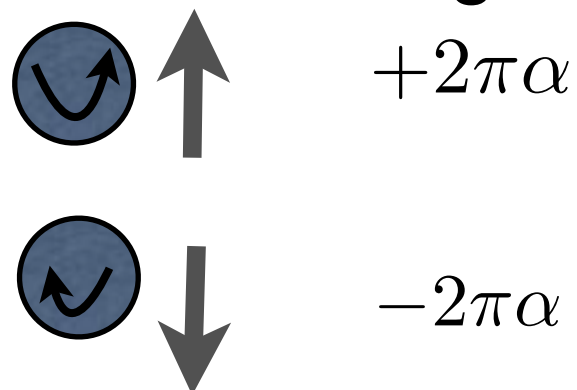


Two resonator case:



$$H_{eff} = -\kappa \hat{a}_{x+1}^\dagger \hat{a}_x e^{-2\pi i \alpha} - \kappa \hat{a}_x^\dagger \hat{a}_{x+1} e^{2\pi i \alpha}$$

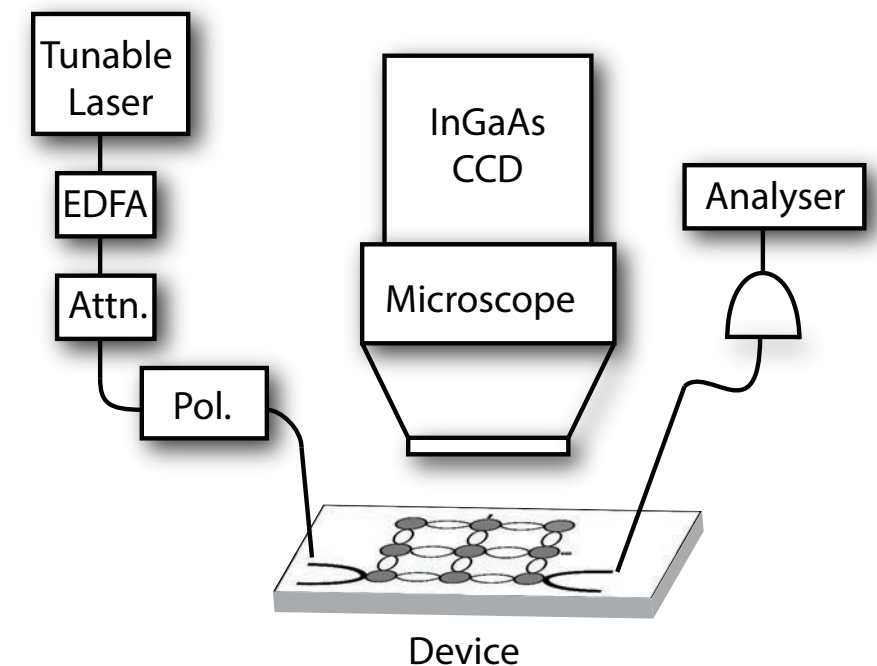
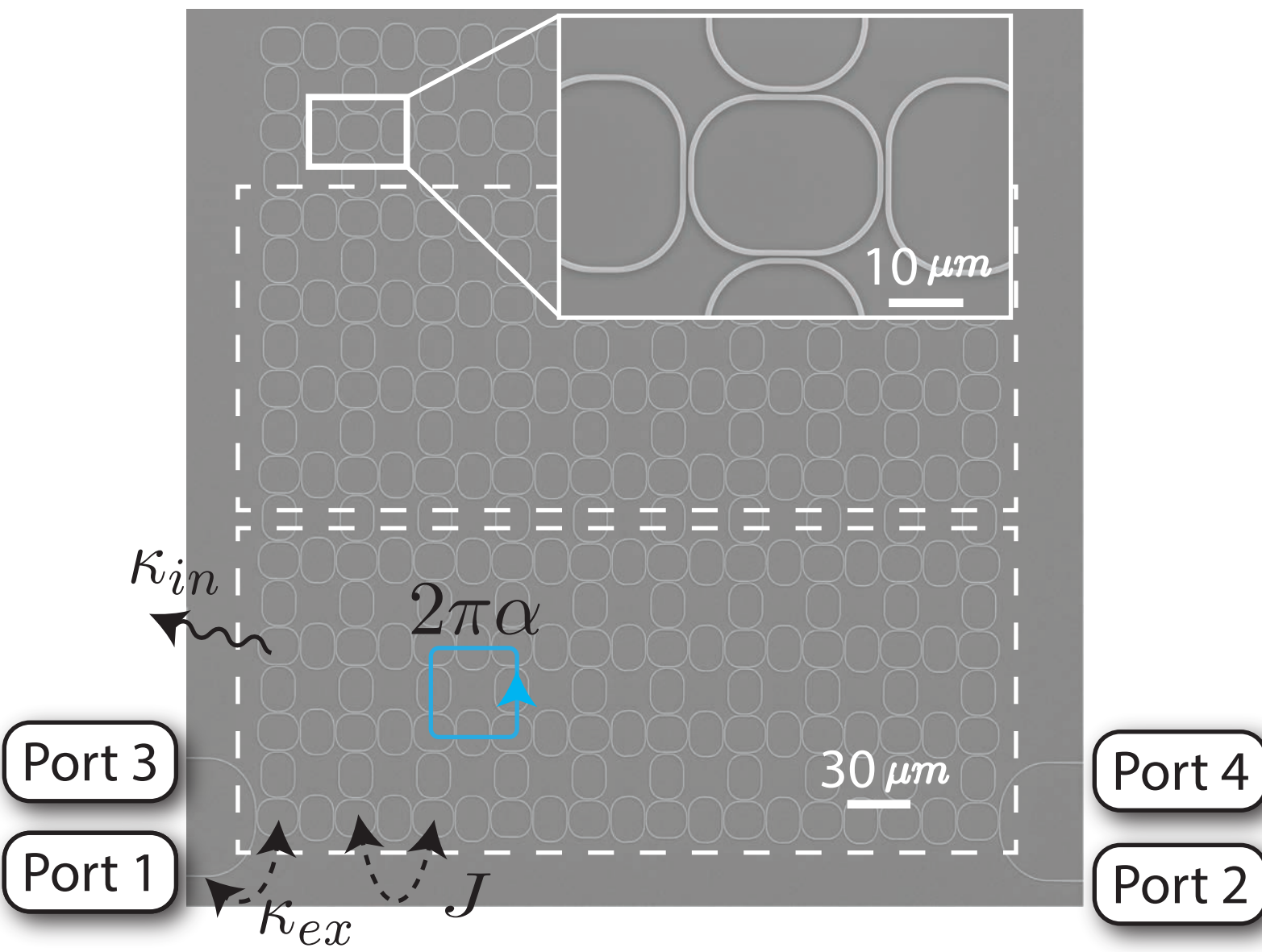
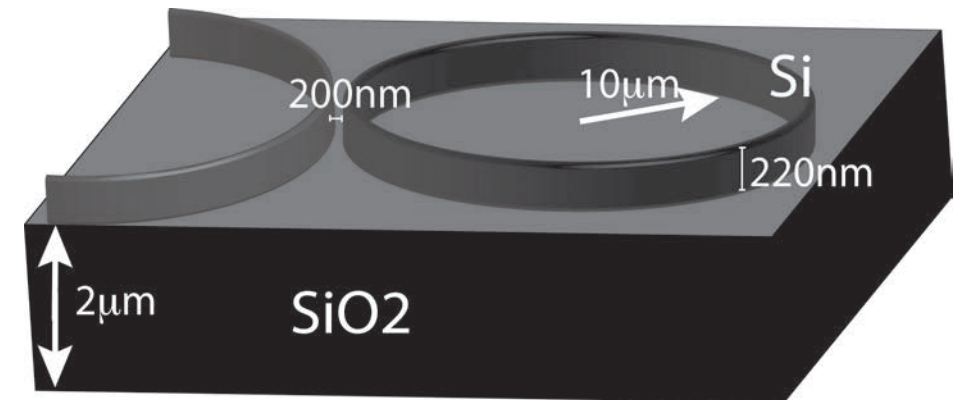
Pseudo Spin Magnetic field



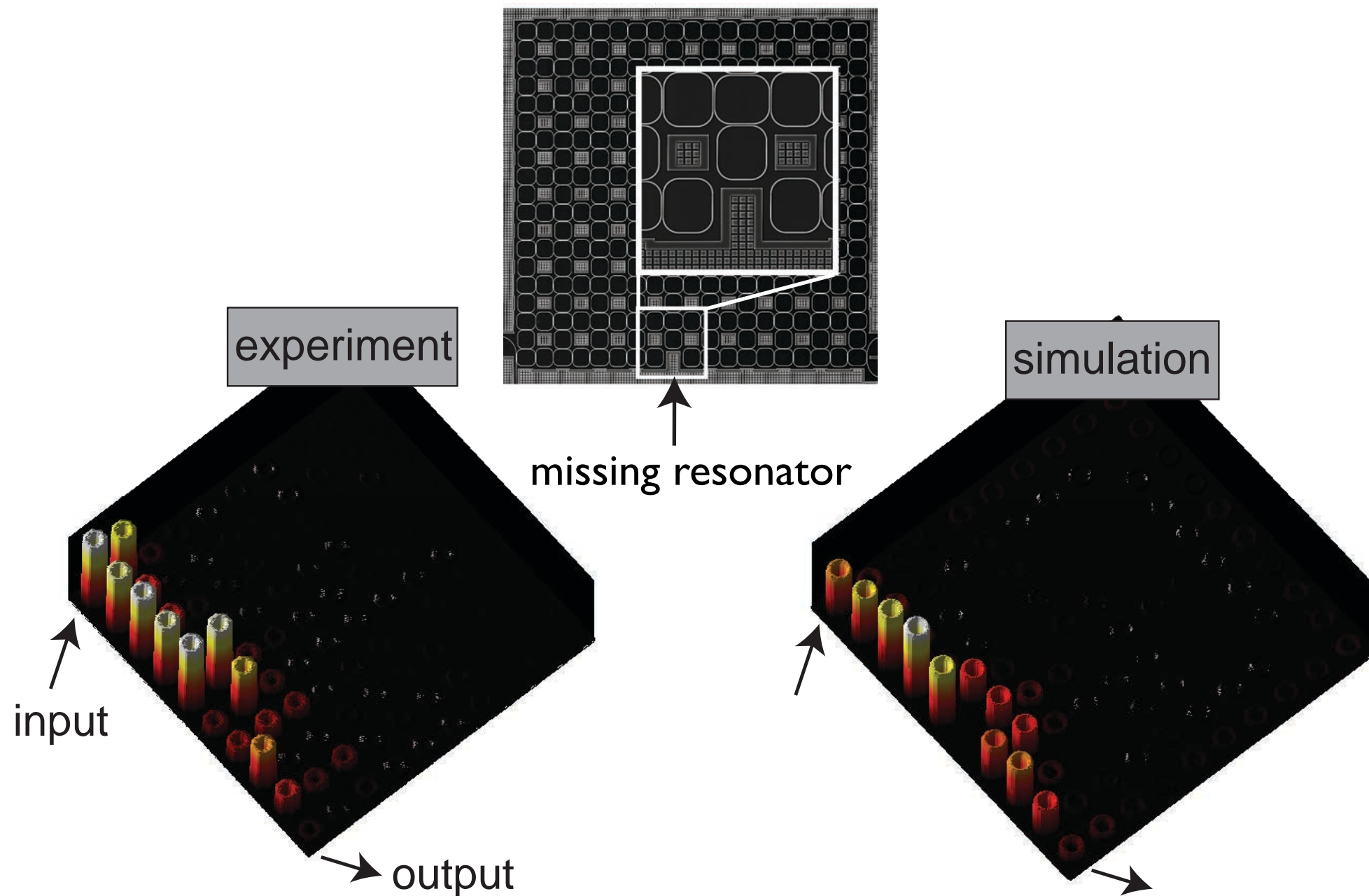
MH, Demler, Lukin, Taylor Nature Physics 7, 907 (2011)
 Earlier work with magnetic field for microwave:
 Haldane PRL (2008), Soljacic et al. Nature (2009)

Experimental realization of the gauge field

Silicon-on-Insulator technology



Robustness against an introduced disorder



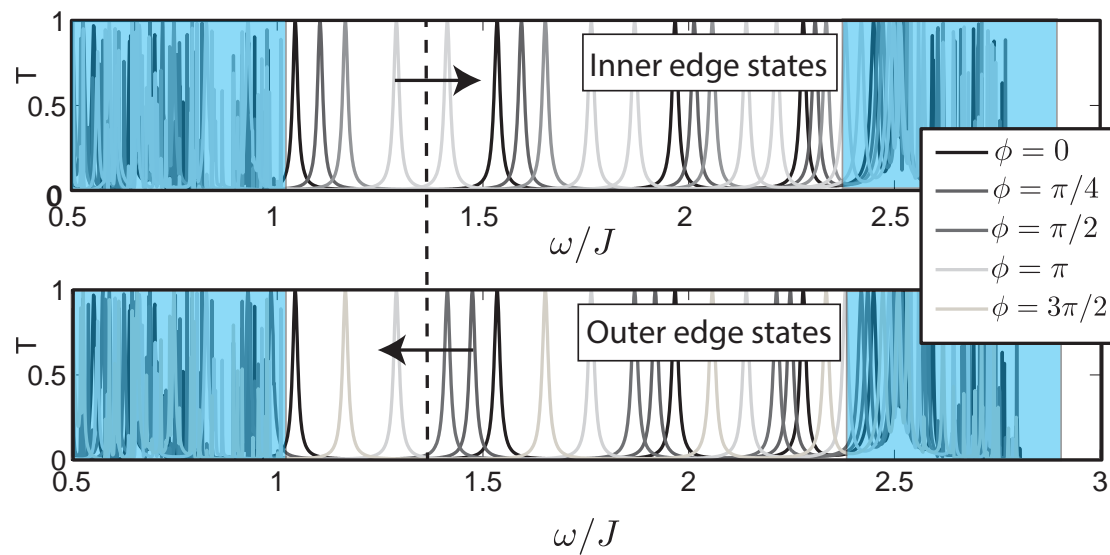
First observation of photonic topological edge states
MH, S. Mittal et al. Nature Photonics 7, 1001 (2013)
Rechtsman et al. Nature (2013)

What's behind this robustness?

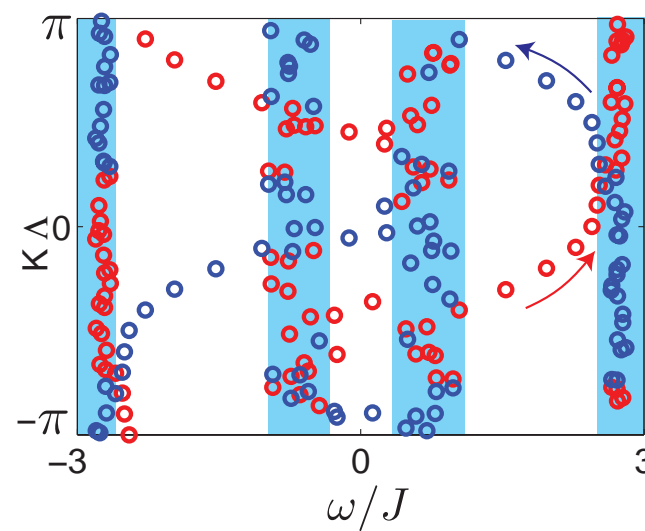
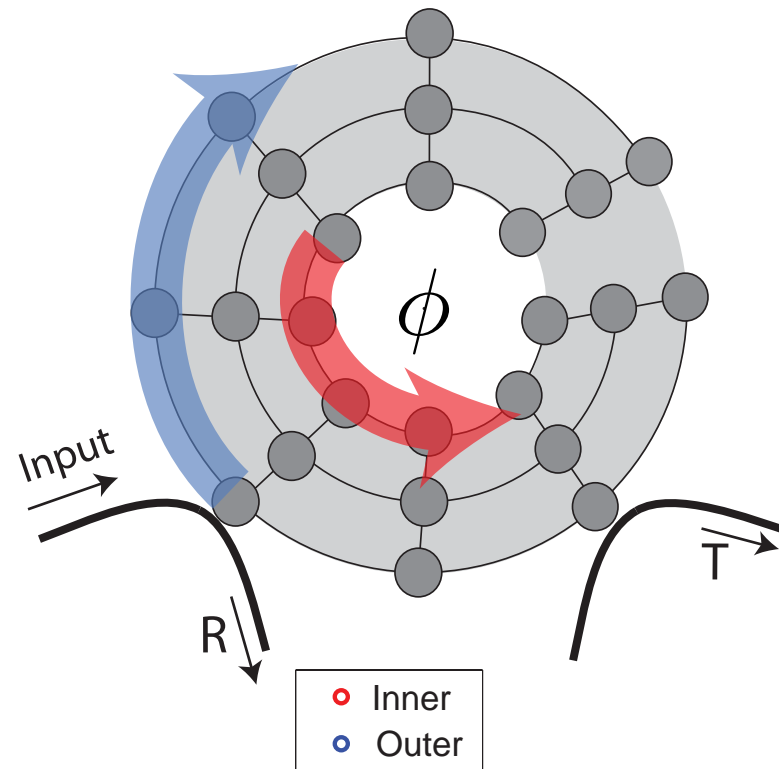
How can one **characterize** and **measure** it?

Topological invariants in photons

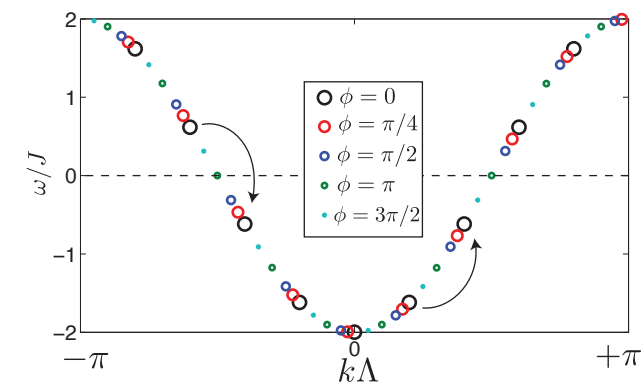
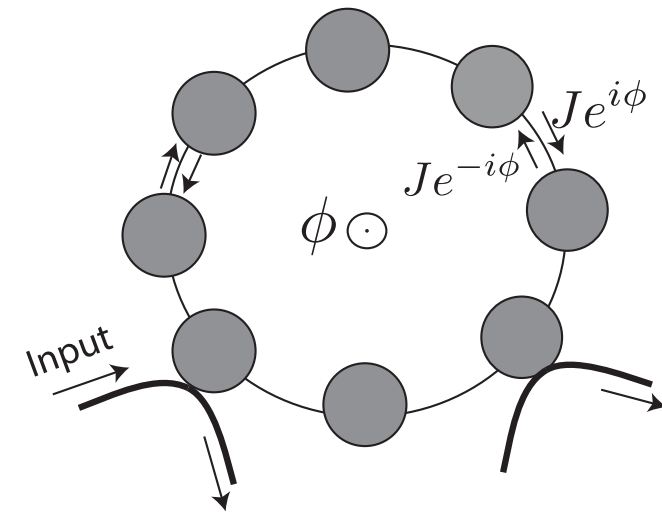
- Quantum Hall effect: Topological orders manifest themselves as integer prefactors in conductance
- What is the manifestation of this integer order in a photonic system?



Laughlin's argument



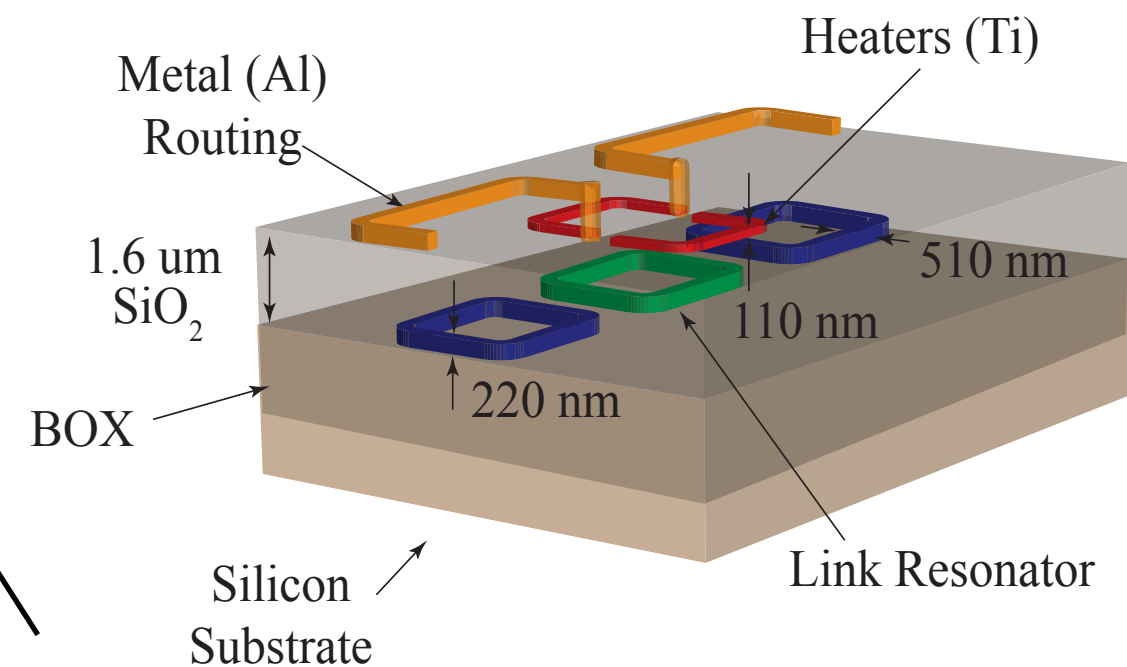
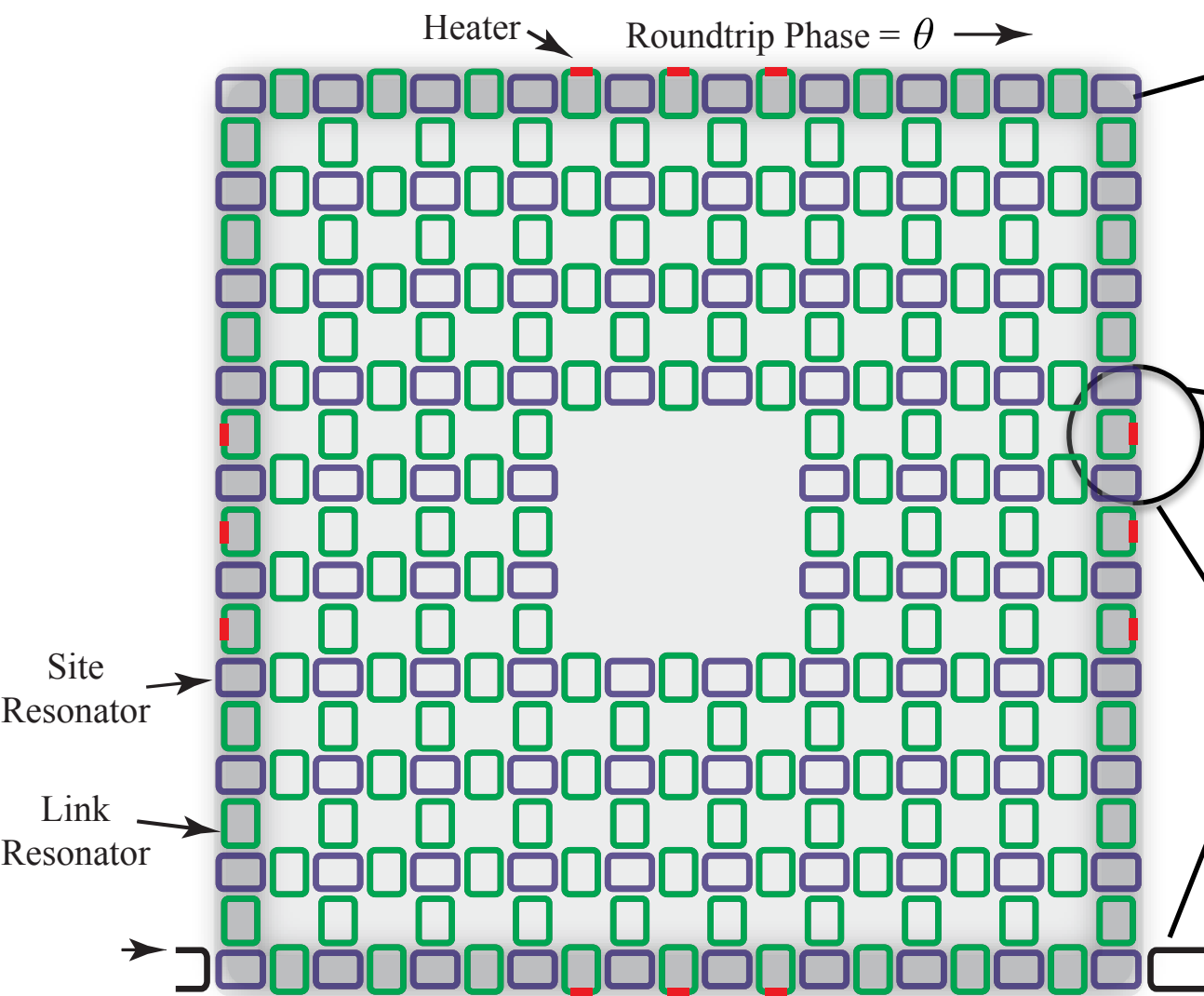
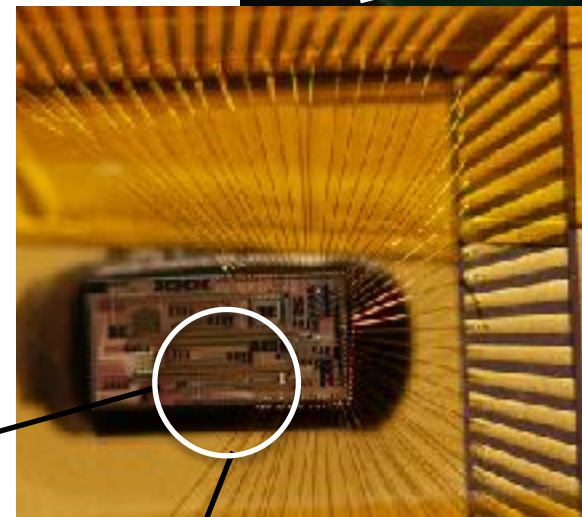
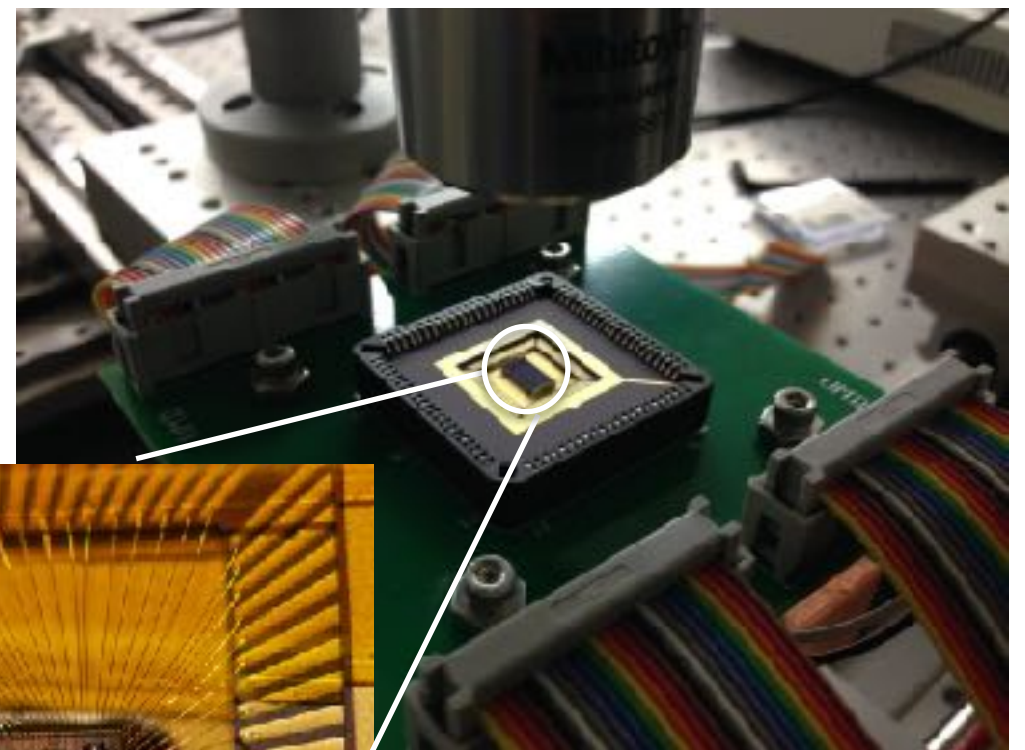
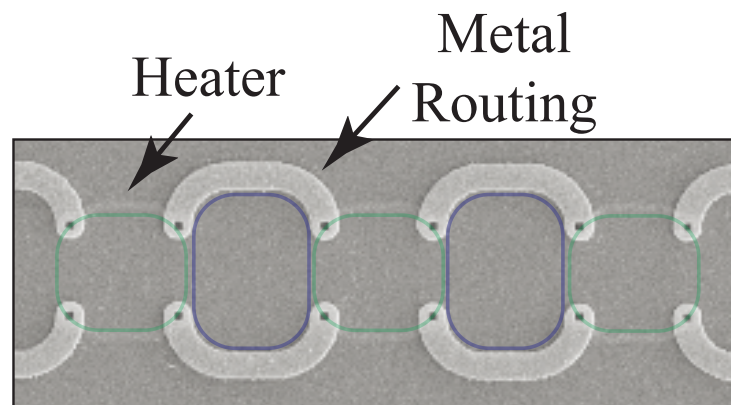
1D ring



MH, PRL 112, 210405 (2014)

see also: Ozawa et al. PRL 2014, Bardyn et al., Y. Chong

Experimental realization



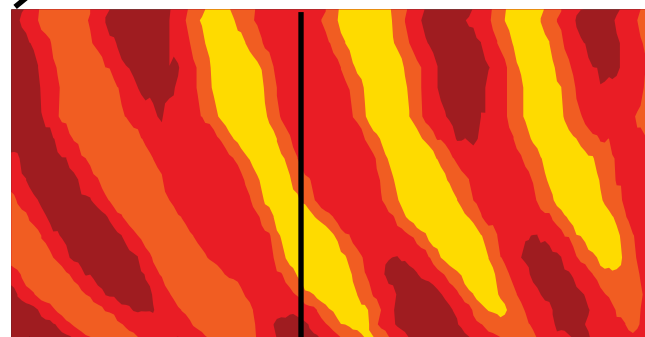
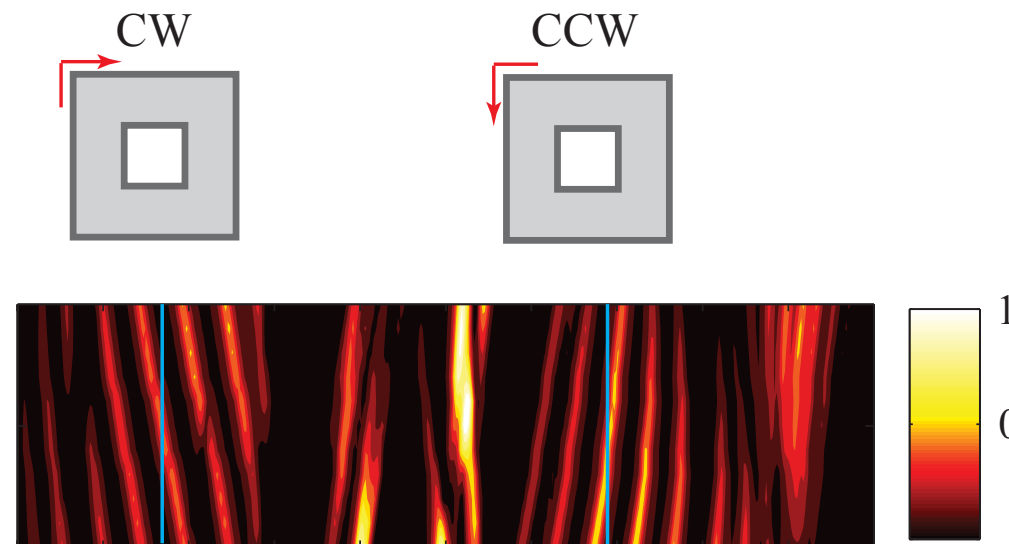
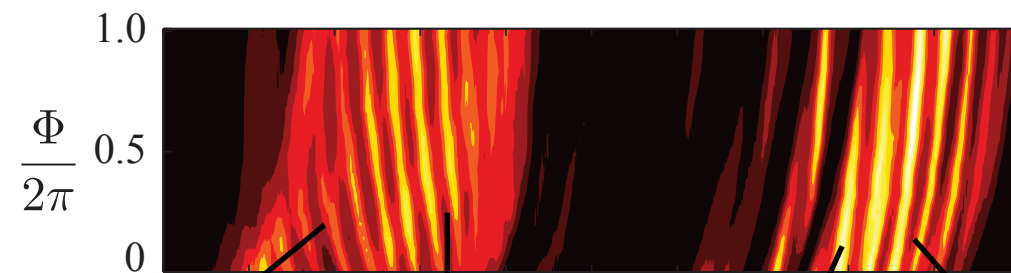
Based on proposal: MH PRL 112, 210405 (2014)

Observation of spectral flow

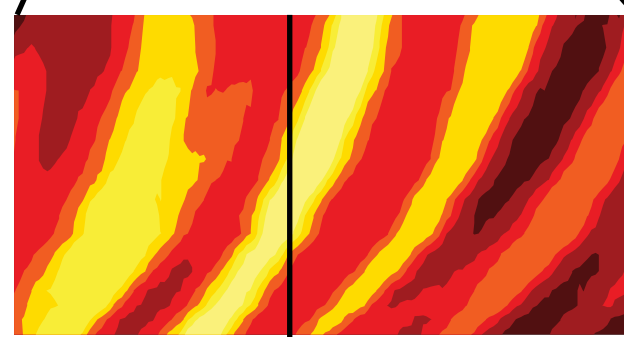
Measured

Simulated

gauge at
outside edge



$k = +1.0(1)$



$k = -1.0(2)$

S. Mittal, S. Ganeshan, J. Fan, A. Vaezi, MH Nature Photonics 10, 180 (2016)

1D work by O. Zilberberg PRL (2013), Y. Chong PRX (2015)

OPTICS & PHOTONICS NEWS



S. Mittal, W. DeGottardi, and M. Hafezi
Optics and Photonics News, 29 (5), 36-43 (2018)

Topological Photonics

T. Ozawa, H. M. Price, A. Amo, N. Goldman, M. Hafezi, L. Lu, M. Rechtsman, D. Schuster, J. Simon, O. Zilberberg, I. Carusotto

arXiv:1802.04173

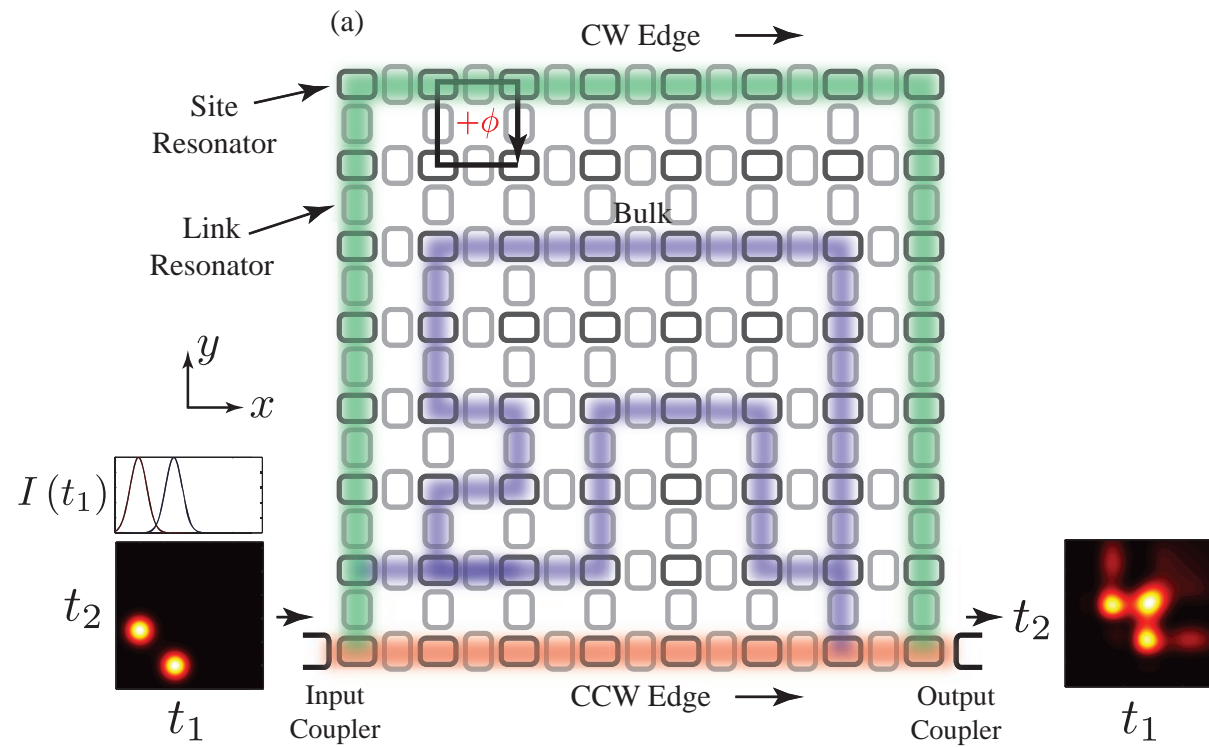
Quantum directions

Linear/weakly nonlinear

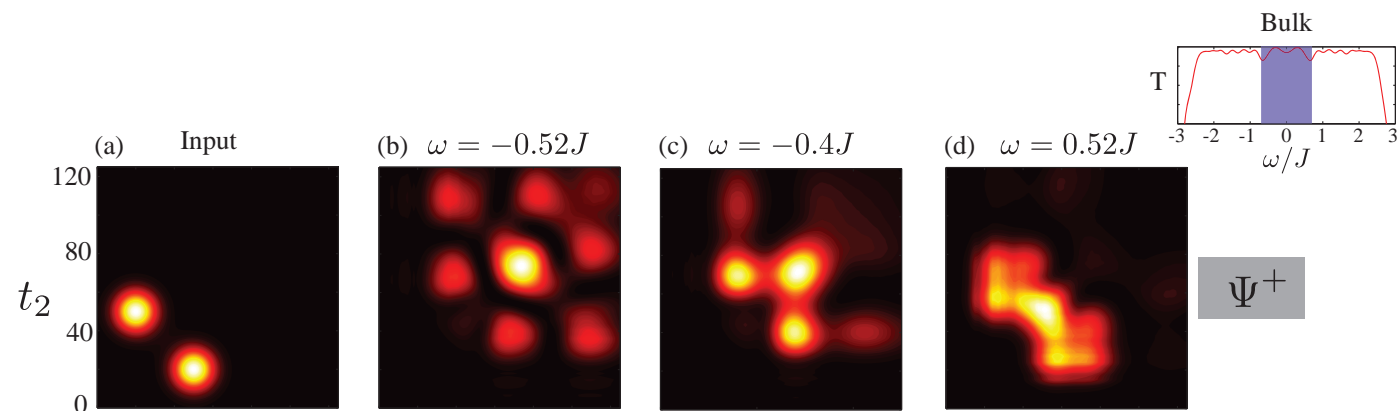
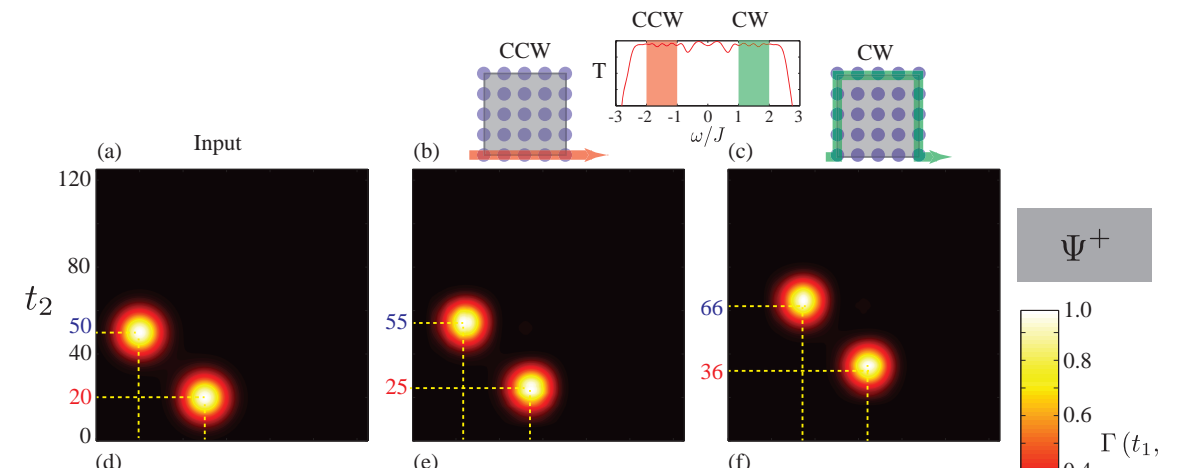
Strong photon-
emitter interaction

Quantum transport of
non-classical light

Quantum transport in topological photonics systems



$$|2\rangle = \int dx_1 dx_2 \psi(x_1, x_2; t) \hat{a}^\dagger(x_1) \hat{a}^\dagger(x_2) |0\rangle$$



Theory: S. Mittal, V. Vikram Orre, and M. H., Optics Express (2016)
see also Rechtsman et al. Optima (2016)

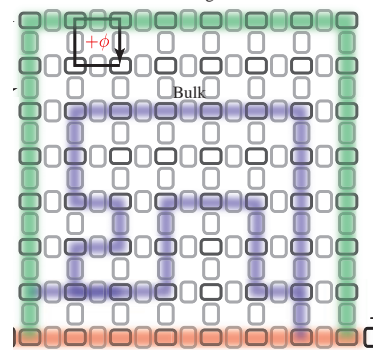
Topological boson sampling?

Quantum directions

Linear/weakly nonlinear

Strong photon-emitter interaction

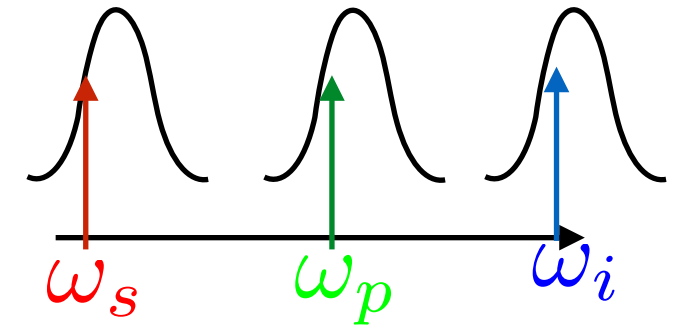
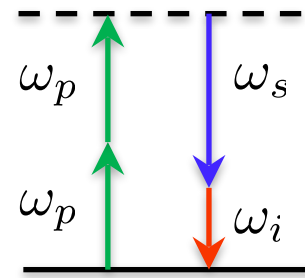
Quantum transport of non-classical light



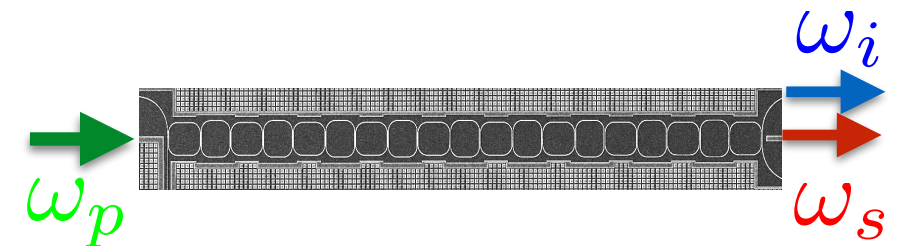
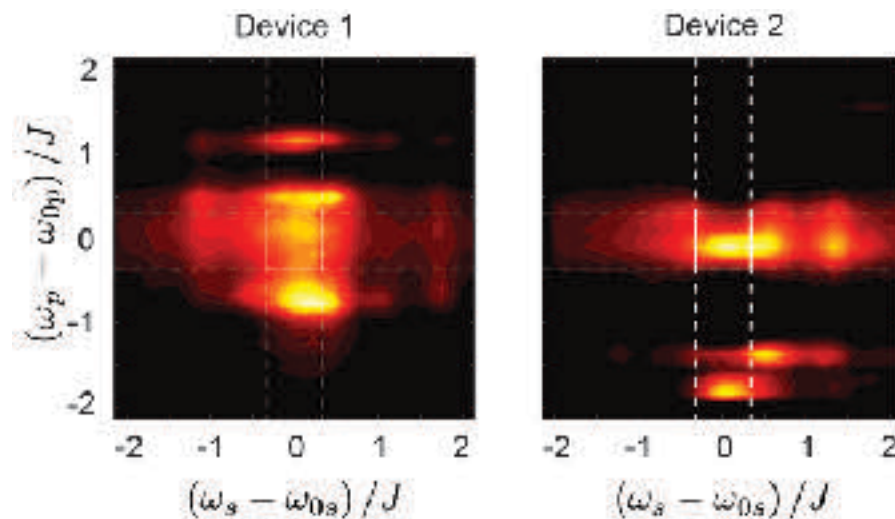
Topologically robust generation and amplification of photons

Photon pair generation

Spontaneous four-wave mixing



$$H_{NL} = \eta \left(a_s^\dagger a_i^\dagger a_p a_p + a_p^\dagger a_p^\dagger a_s a_i \right)$$



1D array of ring resonators
R. Kumar, et. al., Nat. Comm. (2014)
Davanco, et. al., APL. (2014)

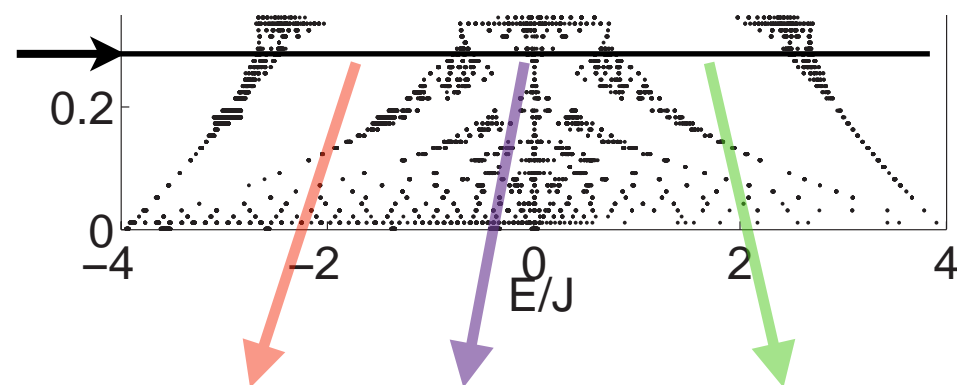
- Photons come at random frequencies
- The spectrum changes from chip to chip

Challenges:

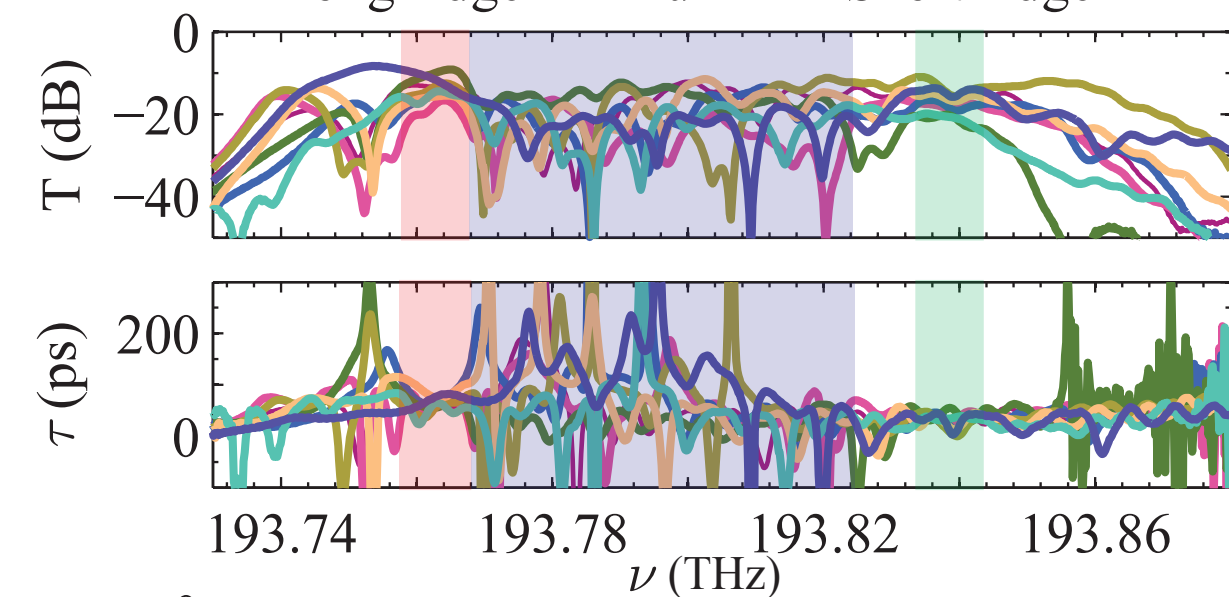
- (1) High **yield**
- (2) High quality (**separability of photons**) $\psi(\omega_i, \omega_s) = \psi_i(\omega_i)\psi_s(\omega_s)$
- (3) Photons should be generated at the **design parameters**

Can topological protection help us?

Transport statistics (classical)

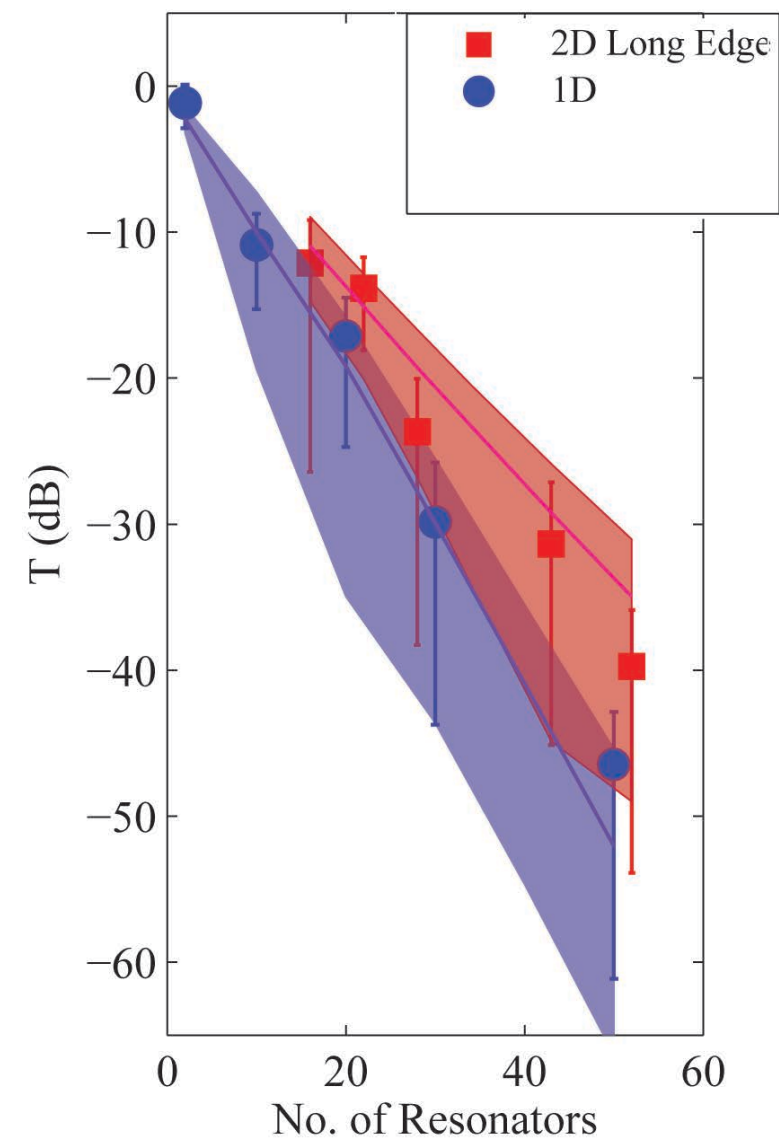


Long Edge Bulk Short Edge

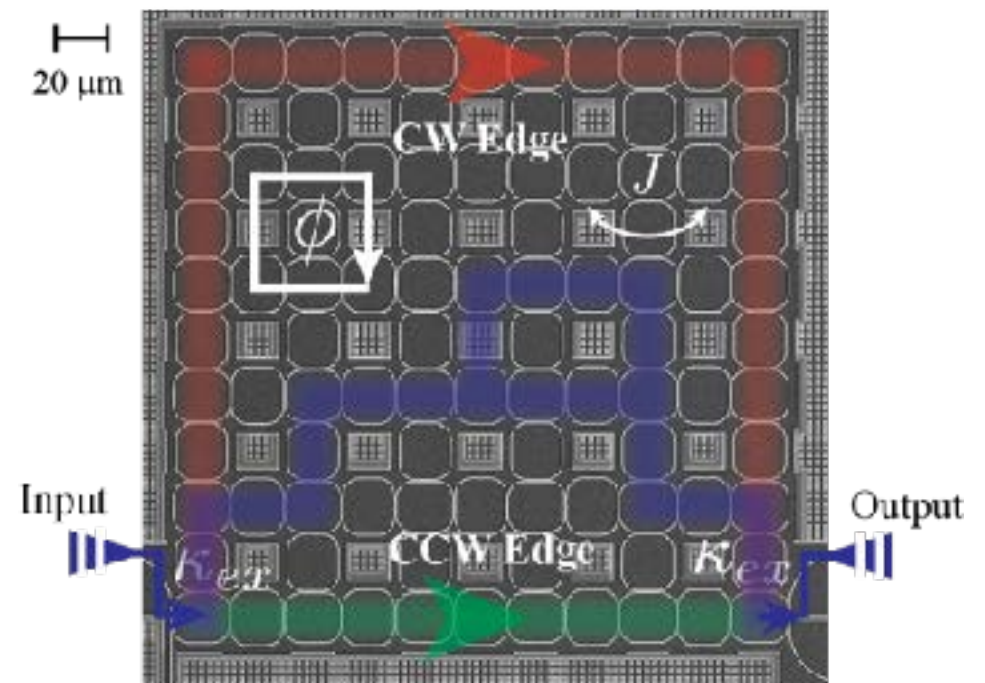
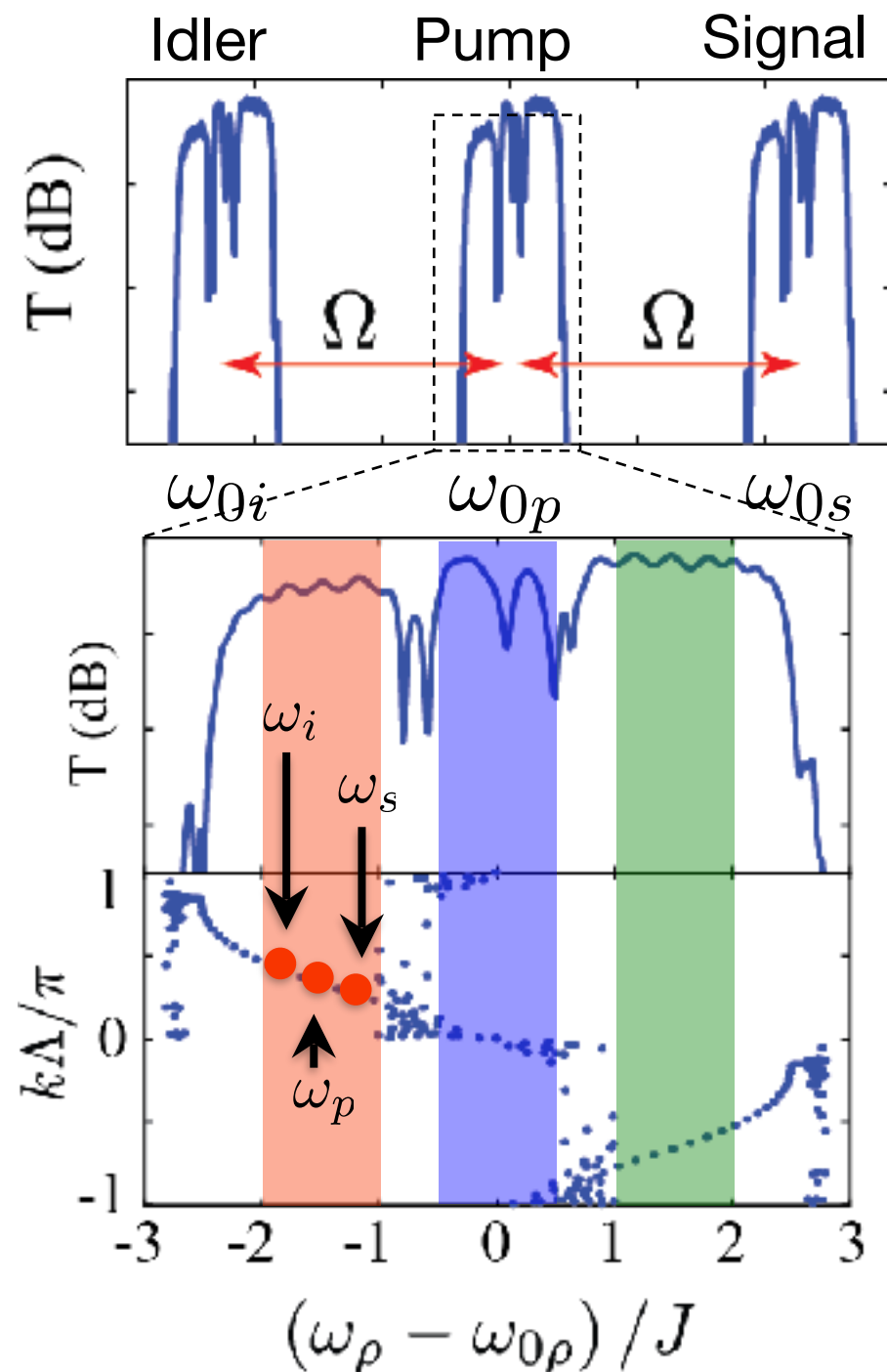


15x15 arrays

Different colors: different samples



Robust generation of photon pairs



- Signal/Idler spectrum confined only to edge
- Linear Dispersion: Enhanced interaction
 $\Delta k \approx 0$
- Robustness against disorder

Fair comparison: trivial/topological

take the same material, fabrication process,
same condition, e.g. gain, pump intensity

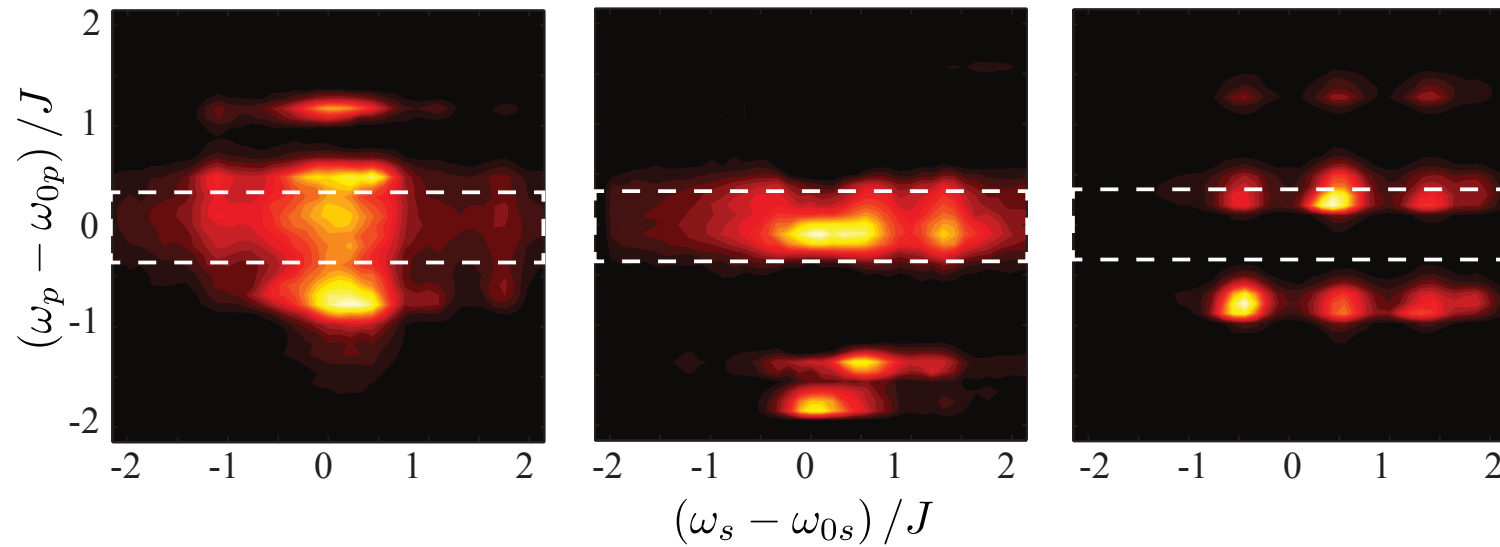
Mittal et. al., arXiv 1709.09984

see also a theory proposal for Topological amplifier
Peano, Houde, Marquardt, Clerk PRX (2016)

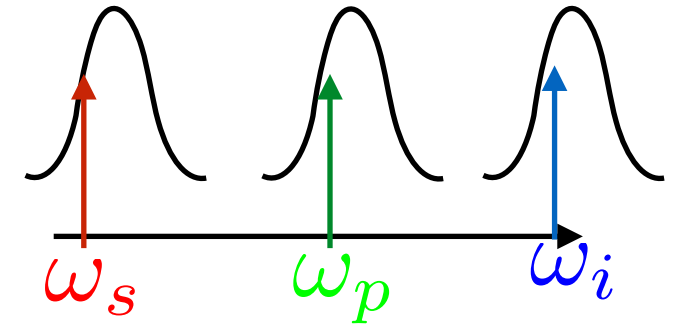
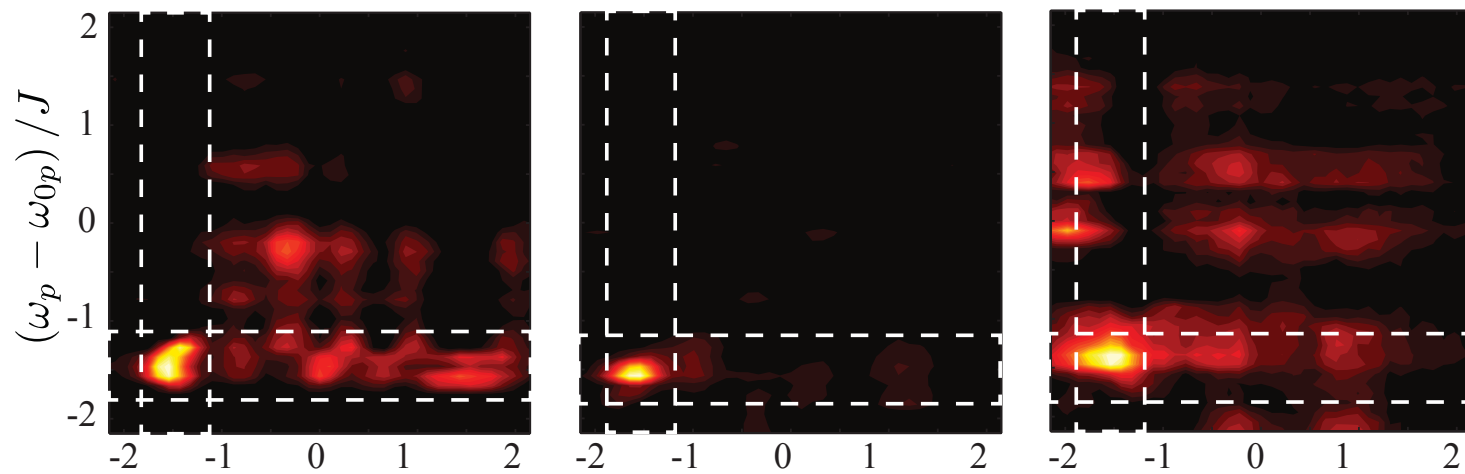
Shi, Cirac, Kimble PNAS (2017)

Comparison between trivial and topological

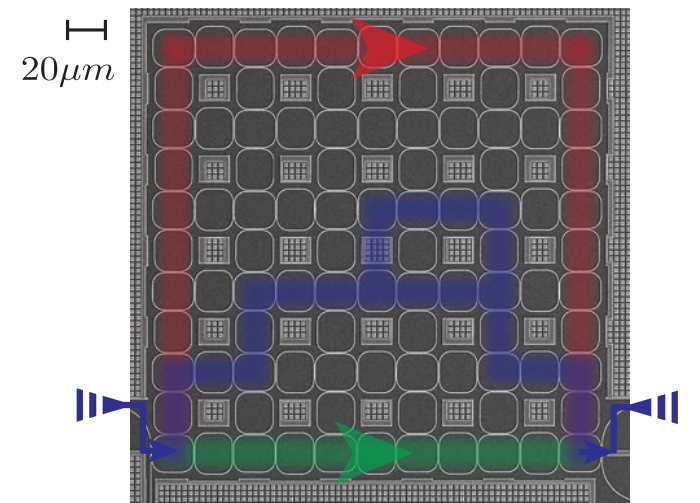
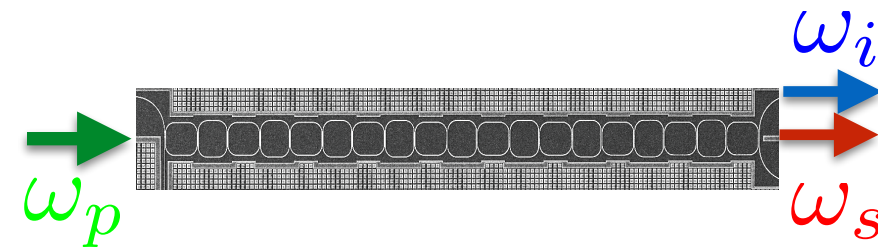
1D



2D



$$H = \Omega^2 a_i^\dagger a_s^\dagger + h.c.$$

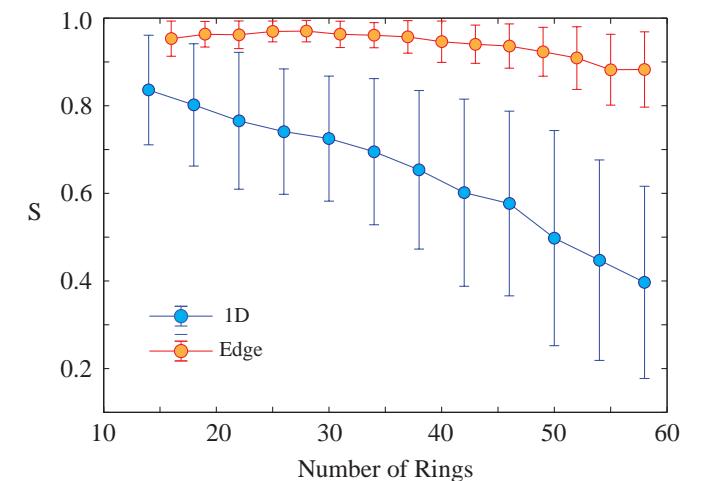


Despite strong disorder

$$\Delta\omega_0 \approx J$$

$$\Delta\phi \approx 0.1$$

2D structure gives a higher quality and reproducible spectra for the same yield

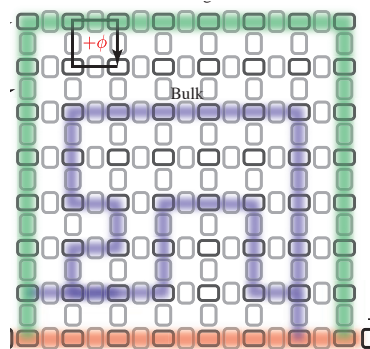


More results are removed due to journal embargo

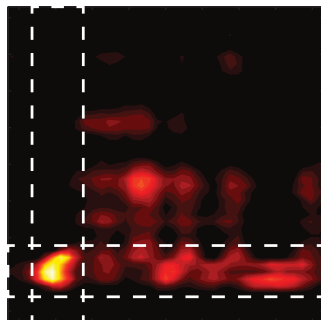
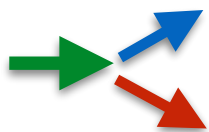
Quantum directions

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Quantum transport of non-classical light

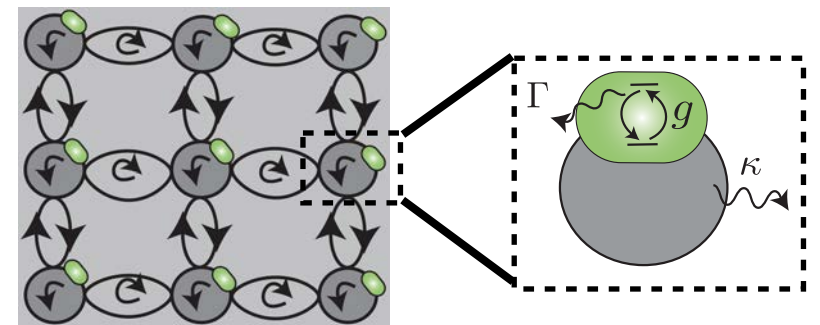


Topologically robust generation of photons



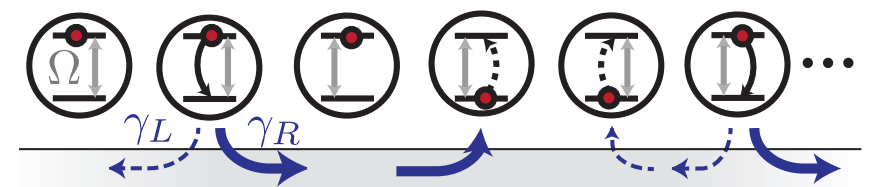
Strong photon-emitter interaction

Fractional Quantum Hall states



MH, Taylor, Lukin NJP (2013)
E. Kapit, MH and S. Simon PRX (2014)
see: Carusotto, Angelakis ...

Chiral Quantum Optics



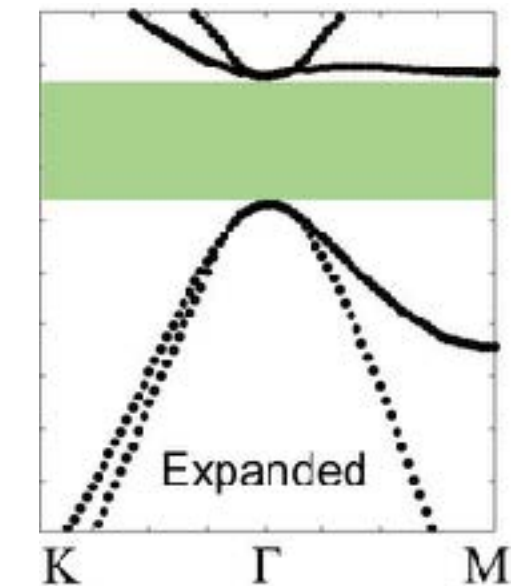
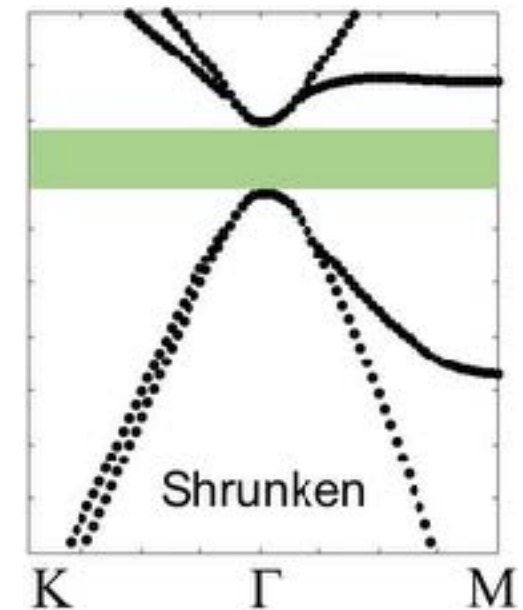
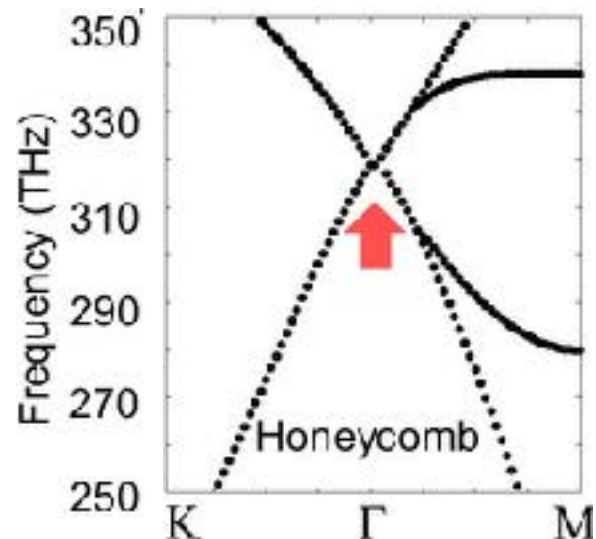
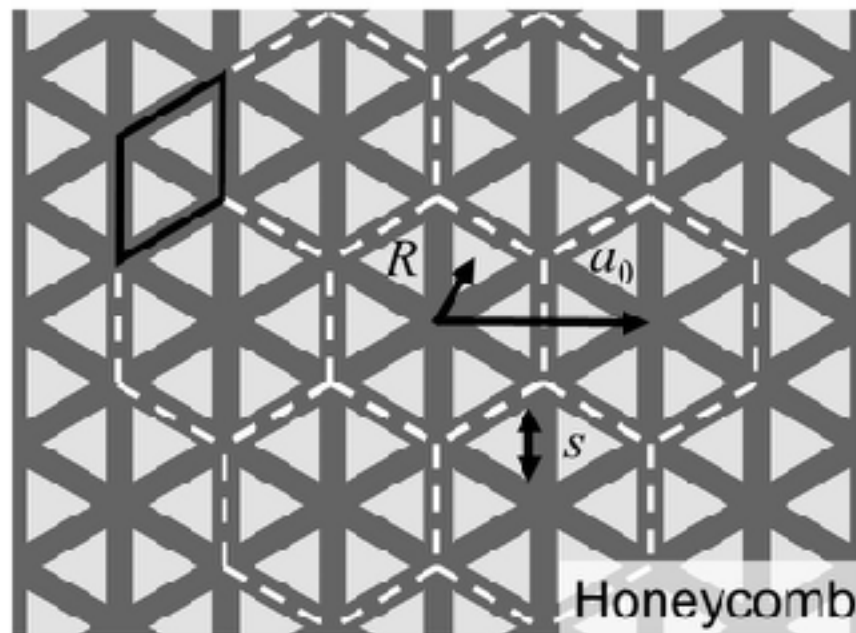
Review: P. Lodahl et al. Nature (2017)

Topological photonic crystals

Goal: A compatible structure with **solid-state emitters**

Challenges:

- ★ **Full bandgap** in the bulk
- ★ E&M field **confined in perp. direction** to the slab



S. Barik et al. NJP (2016)
inspired by Wu and Hu (2015)

Exercise: Jackiw-Rebbi model in 2D

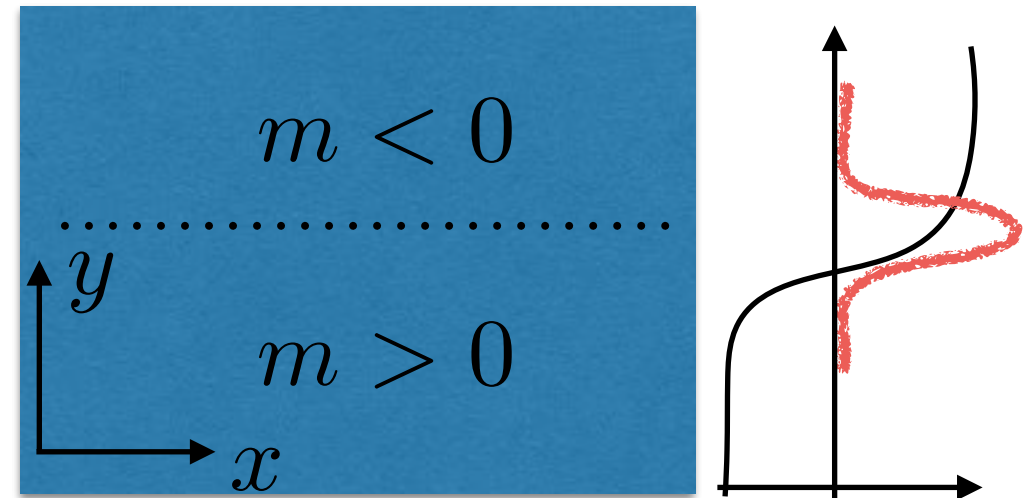
Mass inversion in Dirac equation leads to bound state

$$[-i\hbar v (-\sigma_x \partial_x + \sigma_y \partial_y) + m\sigma_z] \Psi = E\Psi$$

$$m(x, y) = m(y) \text{ and } m(0) = 0$$

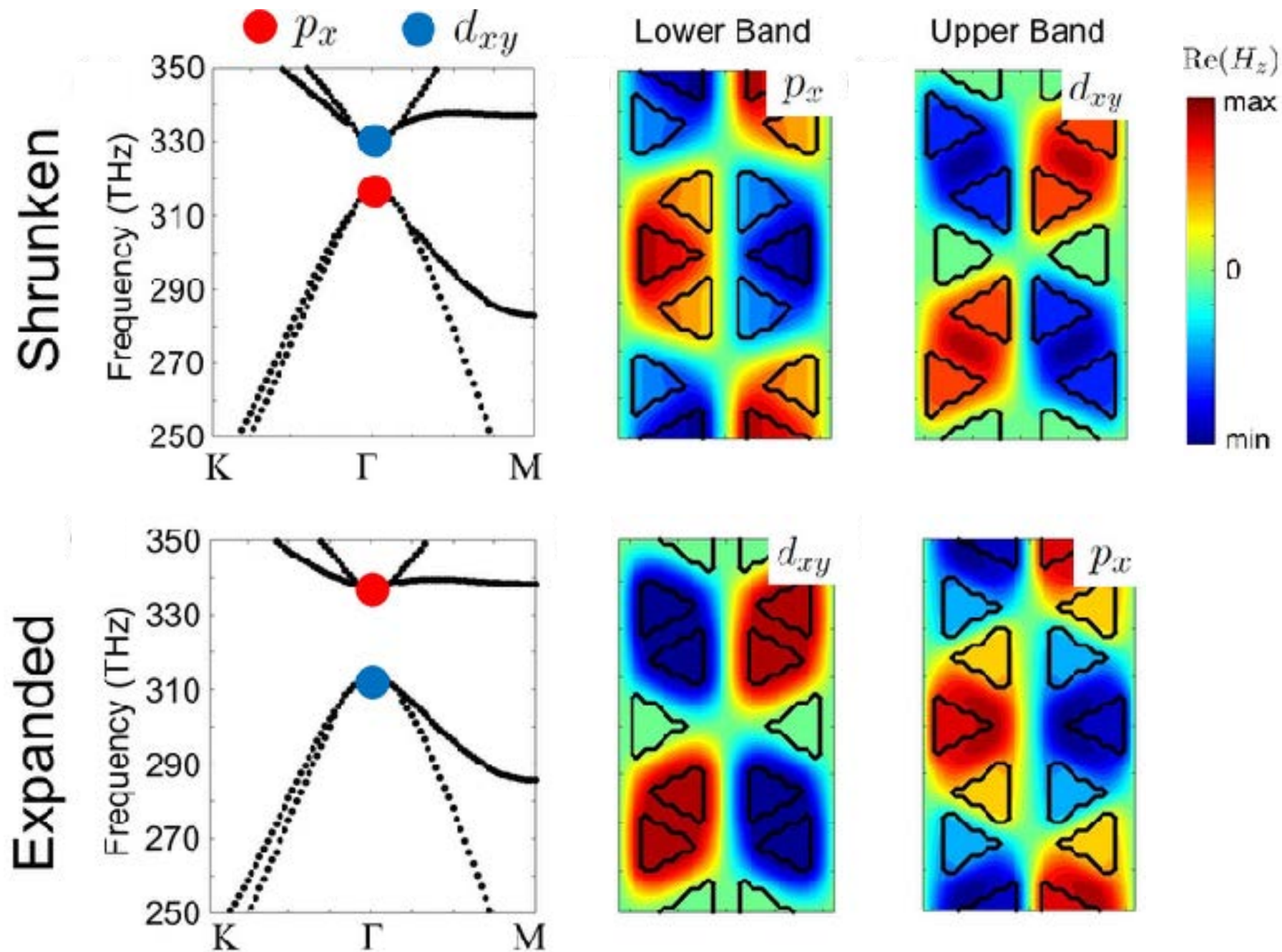
$$\frac{dm}{dy} < 0$$

$$\Psi(x, y) = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \exp\left(\frac{1}{\hbar v} \int_0^y m(y') dy'\right) e^{ik_x x}$$



bound state in y-direction, propagating in x direction

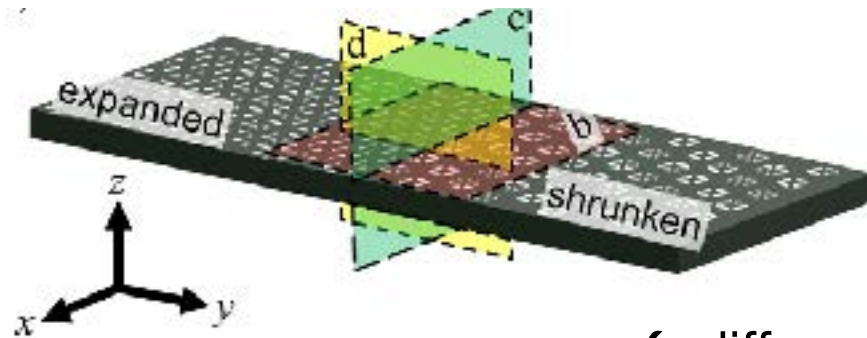
Band inversion: numerical simulation



- ✓ Bulk/edge correspondence: We expect topological edge states to appear at the interface between expanded/shrunk system

helical/chiral topological edge states

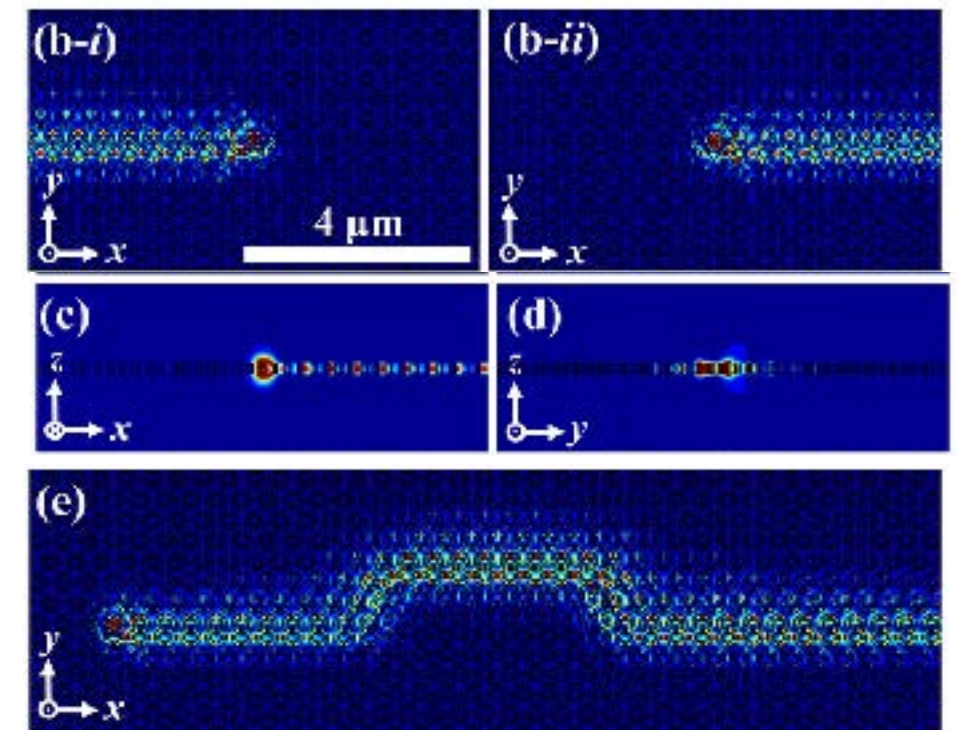
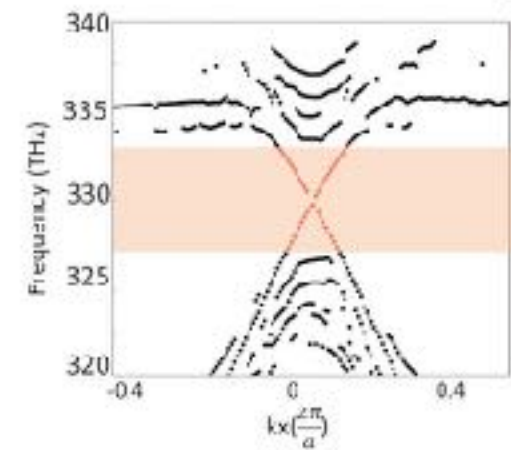
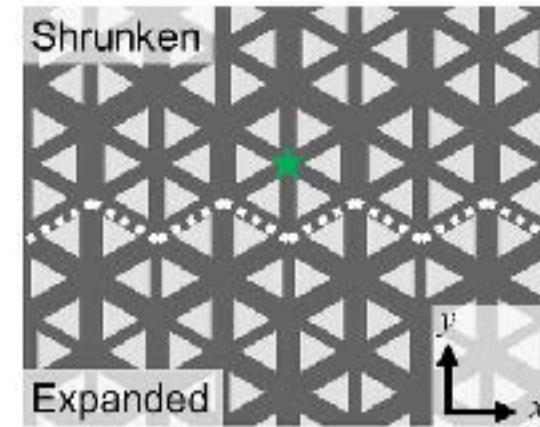
- ✓ Interface between two distinct band structure
- ✓ Topological edge state appear in the bulk gap
- ✓ 2D version/topological version of Lodahl/Rauschenbeutel



- ✓ different polarization propagate in different directions

- ✓ confinement in perp. direction

- ✓ robustness against deformation of edge



PFC collaboration with Edo Waks

Helicity in non-planar waves

$$\vec{E}(\mathbf{r}, t) = \vec{\mathcal{E}}(r) e^{-i\omega t \pm ikz} + c.c.$$

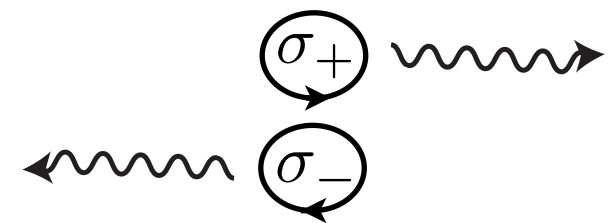
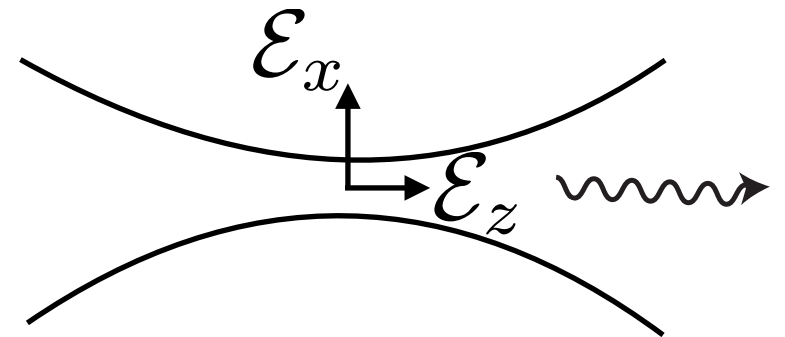
Gauss law:

$$\partial_x E_x + \partial_y E_y + \partial_z E_z = 0$$

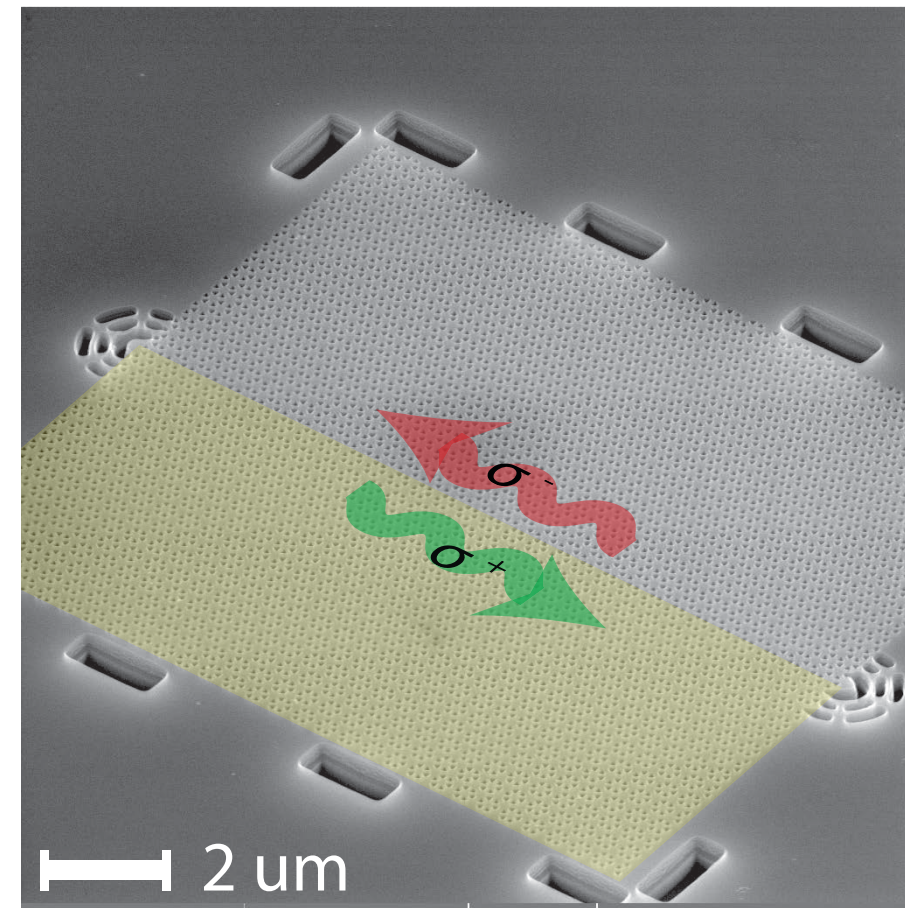
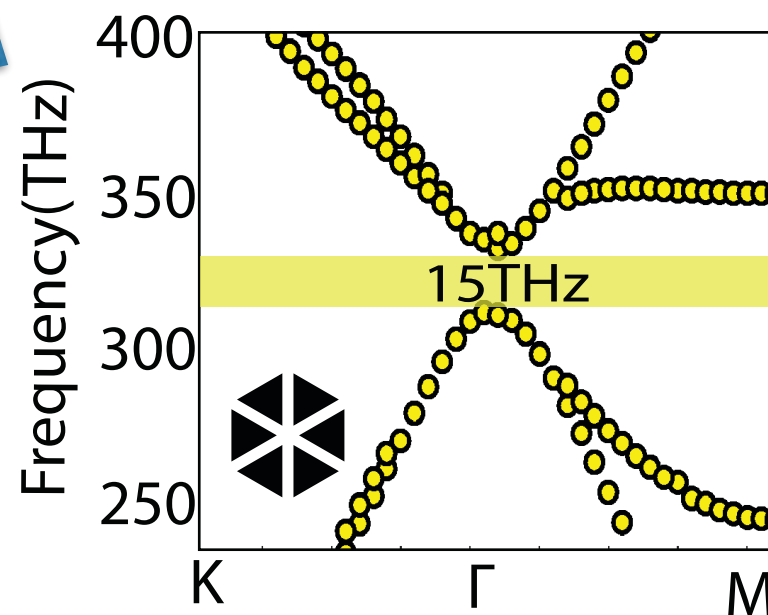
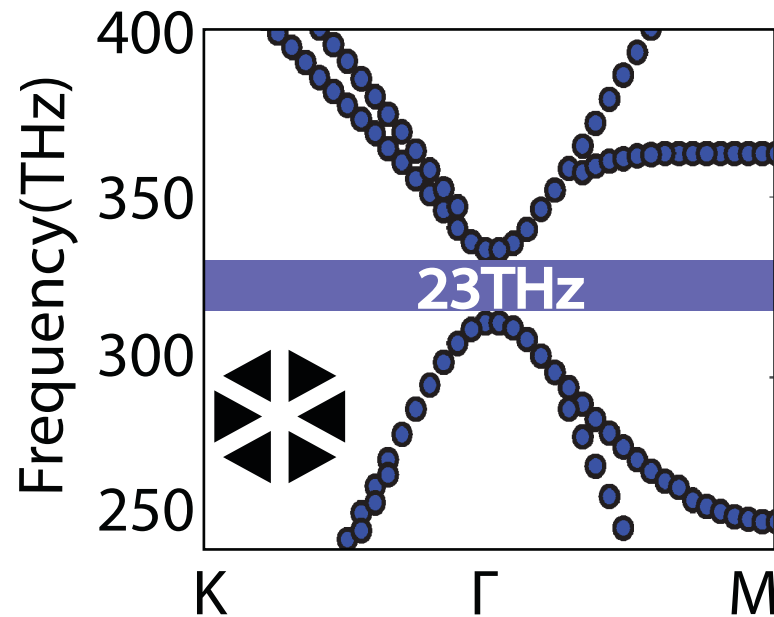
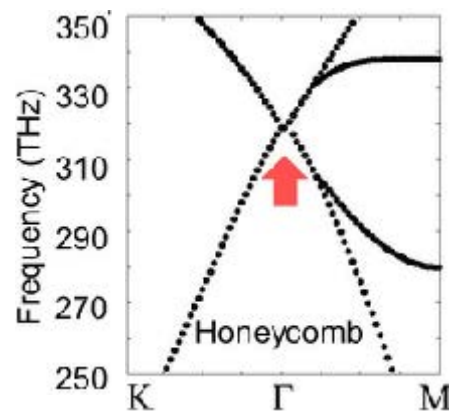
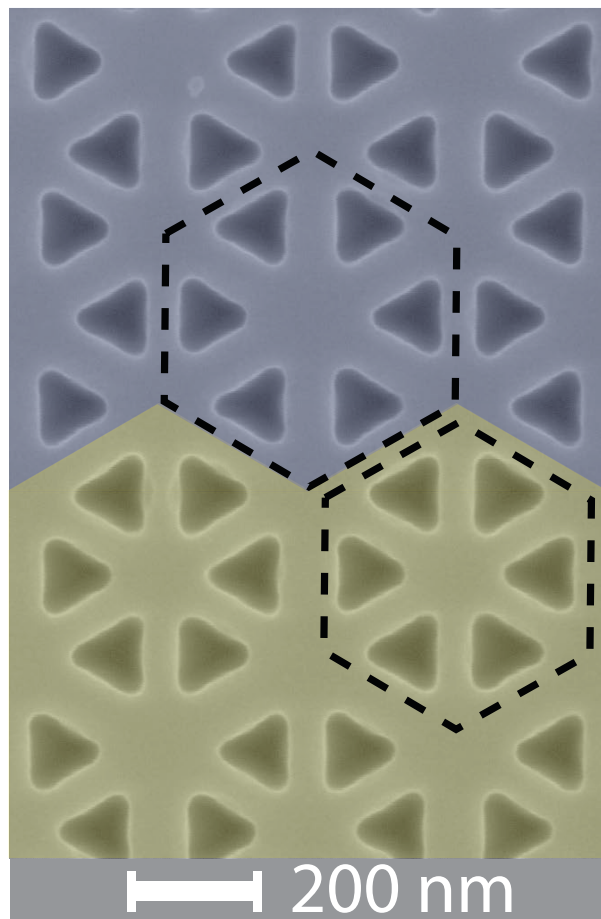
$$\partial_x \mathcal{E}_x + \partial_y \mathcal{E}_y \pm ik \mathcal{E}_z = 0$$

$$\partial_x \mathcal{E}_x \neq 0 \quad \rightarrow \quad \mathcal{E}_z \neq 0$$

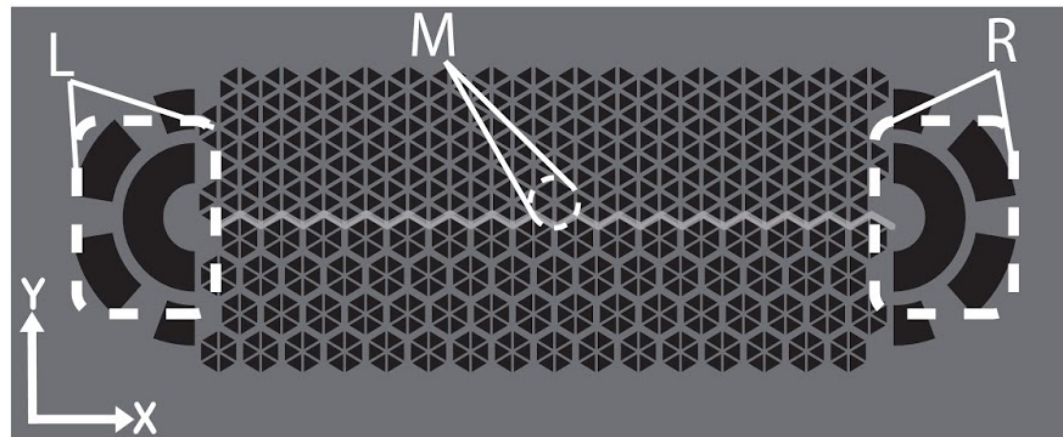
$(\mathcal{E}_x, \mathcal{E}_z)$ are $\pi/2$ out of phase



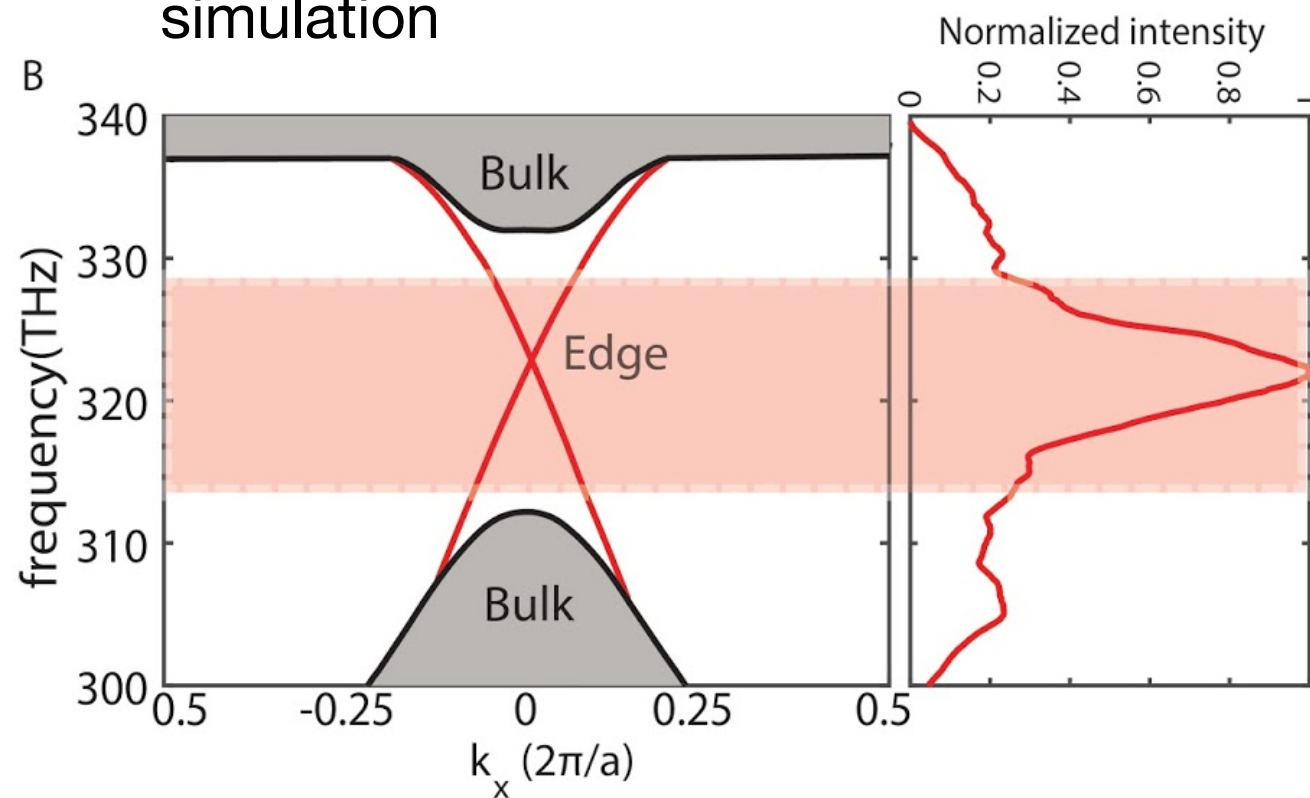
Experimental realization of Topological photonic crystals



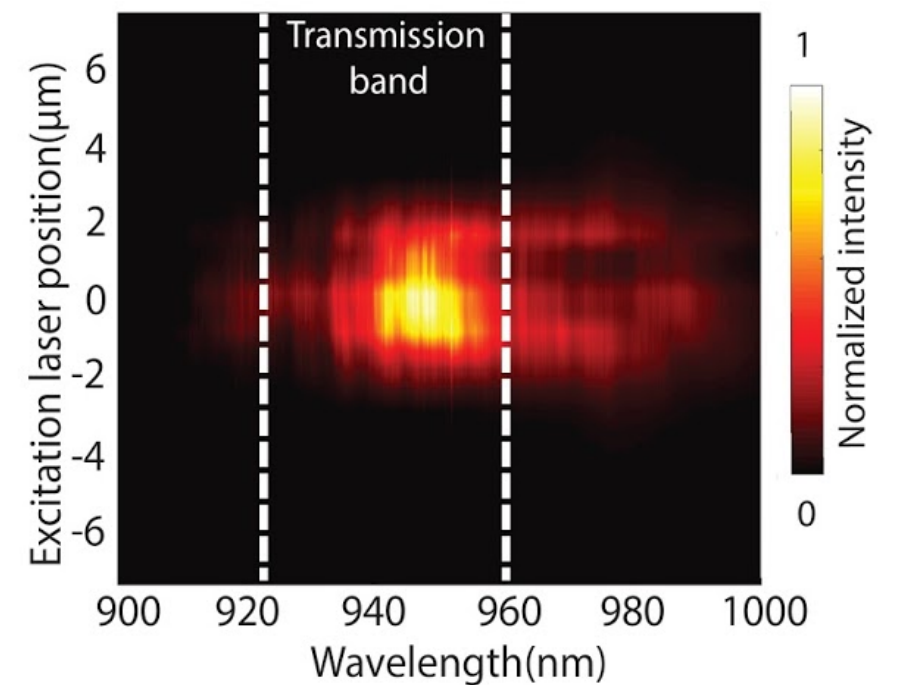
Transmission characteristics of the topological waveguide



simulation

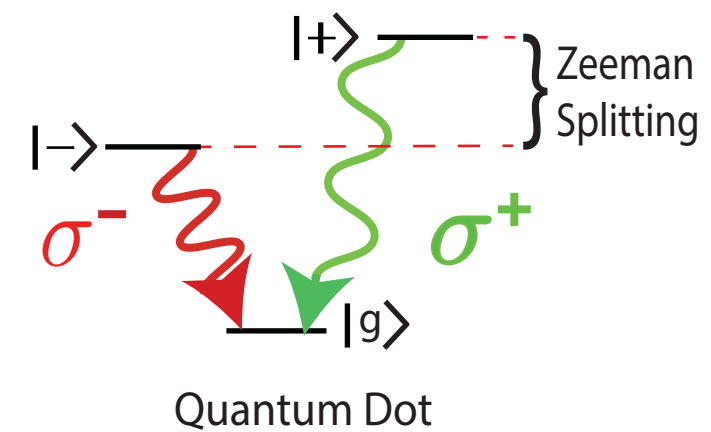
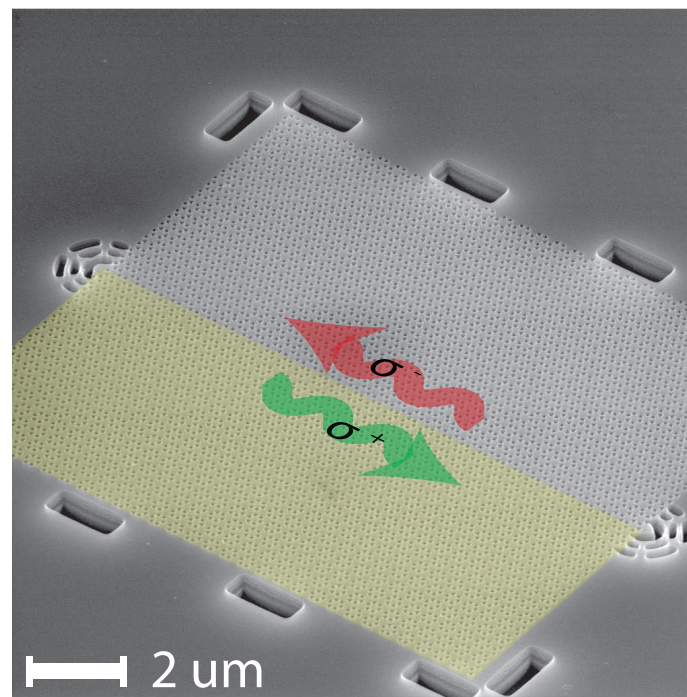


experiment

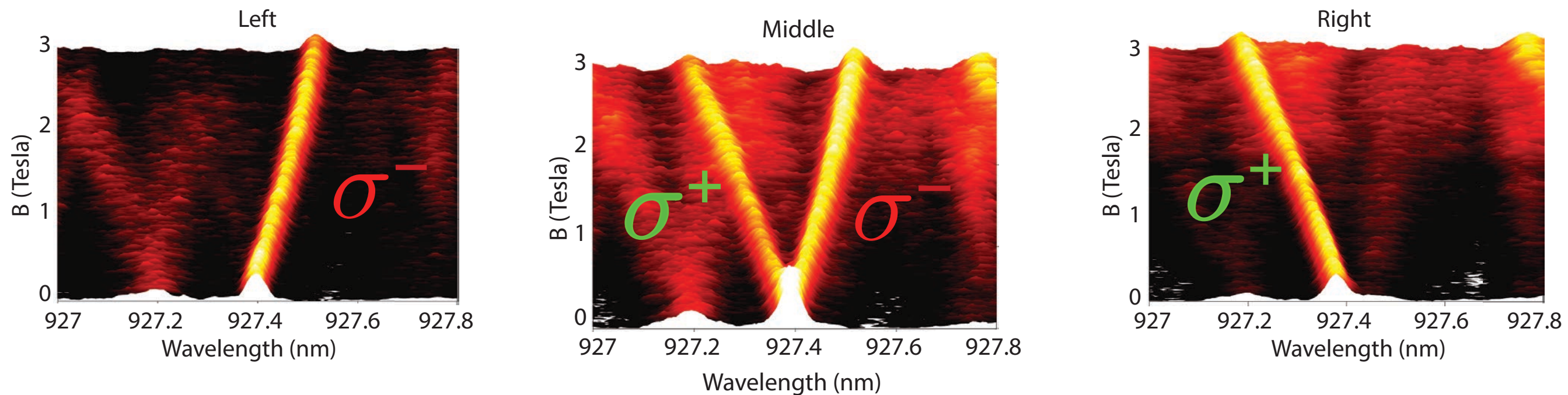


780nm excitation with 1.3 μW power

Chiral topological emission

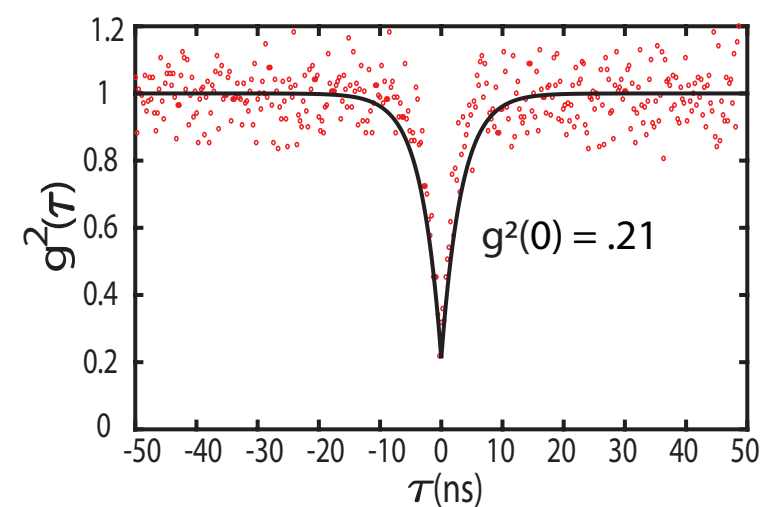
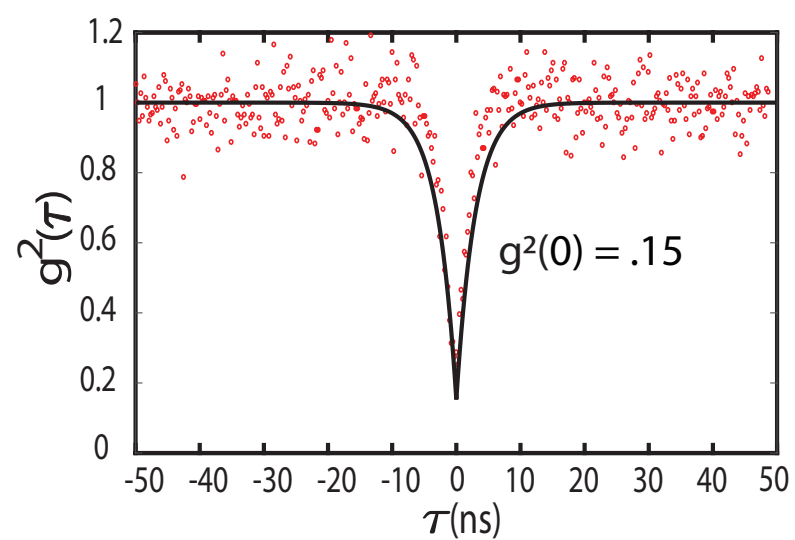
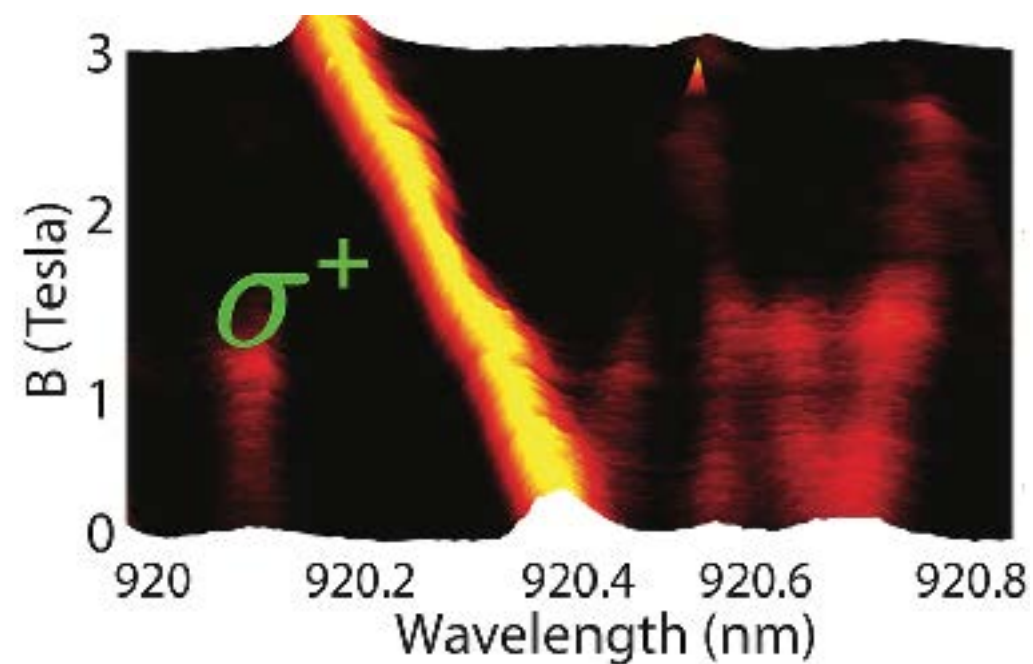
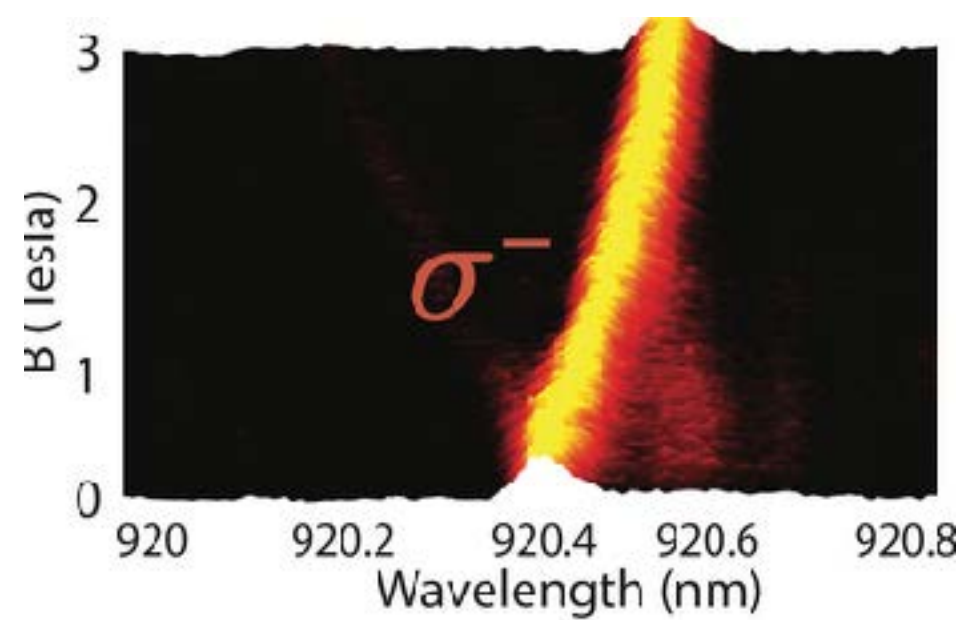
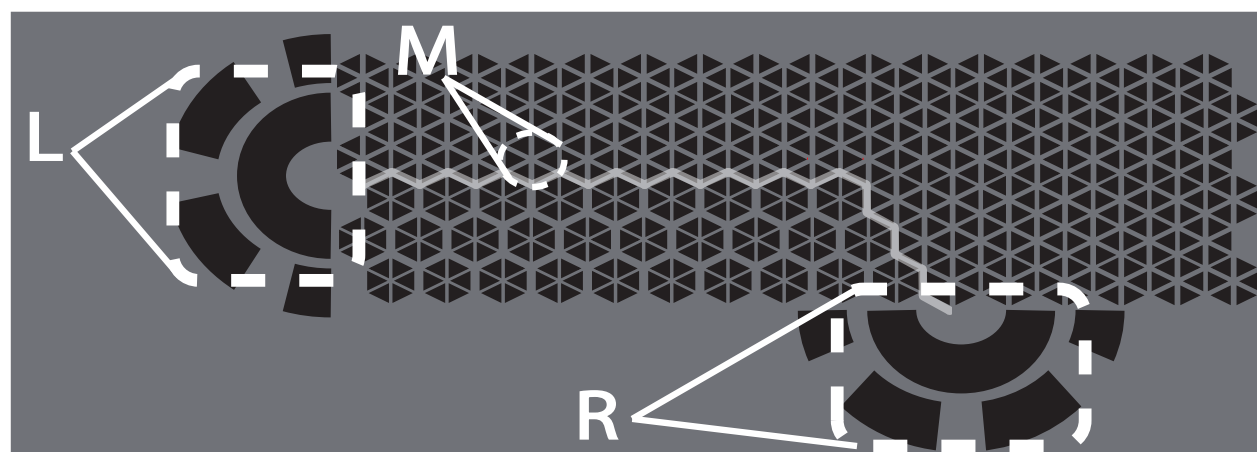


Compare with 1D:
Lodahl/Rauschenbeutel/Zoller Nature (2017)



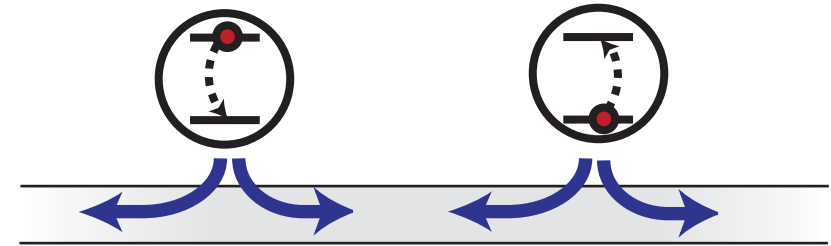
well below the saturation power (10 nW)

Robustness against bend



Outlook: Chiral quantum optics

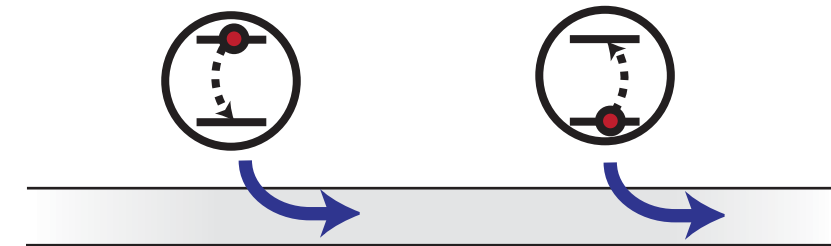
$$\dot{\rho} = -i[H_{\text{sys}} + \gamma \sin(k|x_1 - x_2|)(\sigma_1^+ \sigma_2^- + \sigma_2^+ \sigma_1^-), \rho] \\ + 2\gamma \sum_{i,j=1,2} \cos(k|x_i - x_j|)(\sigma_i^- \rho \sigma_j^+ - \frac{1}{2}\{\sigma_i^+ \sigma_j^-, \rho\})$$



$$\dot{\rho} = \mathcal{L}\rho \equiv -i(H_{\text{eff}}\rho - \rho H_{\text{eff}}^\dagger) + \sigma\rho\sigma^\dagger$$

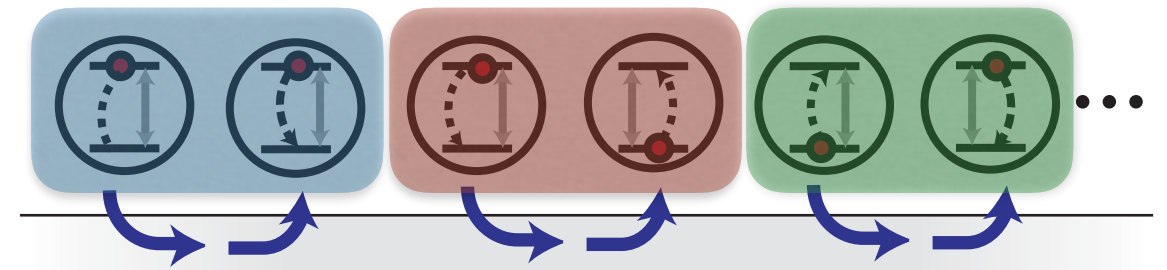
$$H_{\text{eff}} = H_{\text{sys}} - i\frac{\gamma}{2}(\sigma_1^+ \sigma_1^- + \sigma_2^+ \sigma_2^- + 2\sigma_2^+ \sigma_1^-)$$

$$\sigma = \sigma_1^- + \sigma_2^-$$

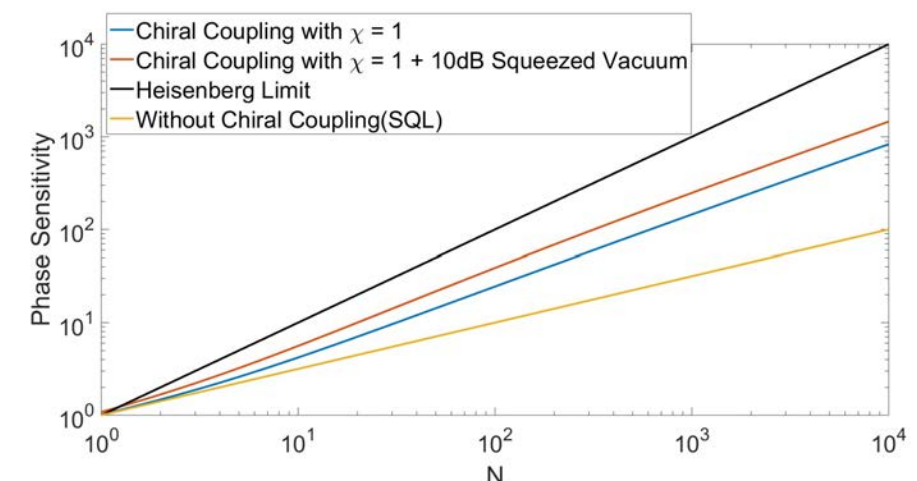
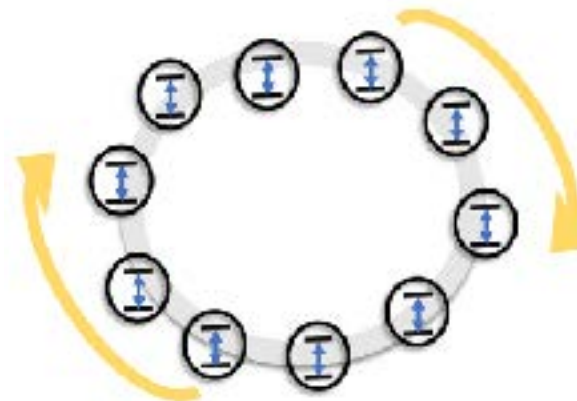


- location independent coupling

$$\Omega(\sigma + \sigma^\dagger)$$



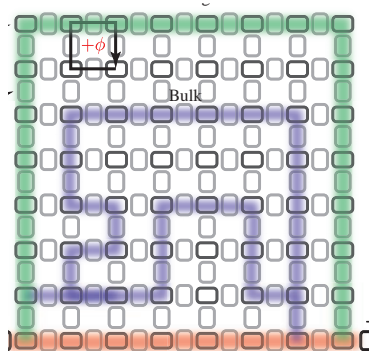
- Chiral coupling overcomes the inhomogeneity of emitter locations: large entanglement
- Topology provides an added robustness



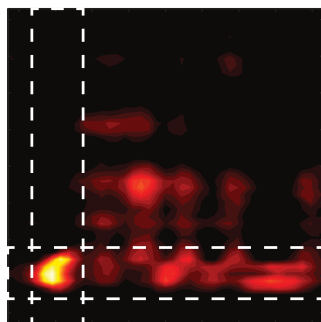
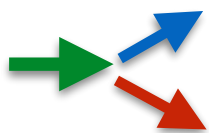
Quantum directions

Linear/weakly nonlinear

Quantum transport of
non-classical light



Topologically robust
generation of photons

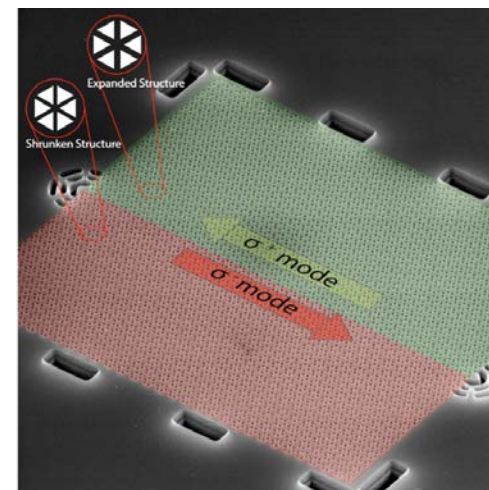


S. Mittal, Vikram Orre, and M. H., OP. EXP. 24, 15632 (2016)
Topologically robust generation of photon pairs (1709.09984)

Strong photon-
emitter interaction

Fractional Quantum
Hall states

Chiral Quantum Optics



S. Barik A. Karasahin C. Flower

New Journal of Physics (2016)
Science, 359, 666 (2018)

Outline of this talk

- Quantum results on topological photonics:
 - Topologically robust generation of photons
 - Topological photonic crystals
- Photons and electronic quantum Hall states
 - Optical probe of IQHE states
 - Driven FQH states and bilayer physics



M. Gullans
(JQI-> Princeton)



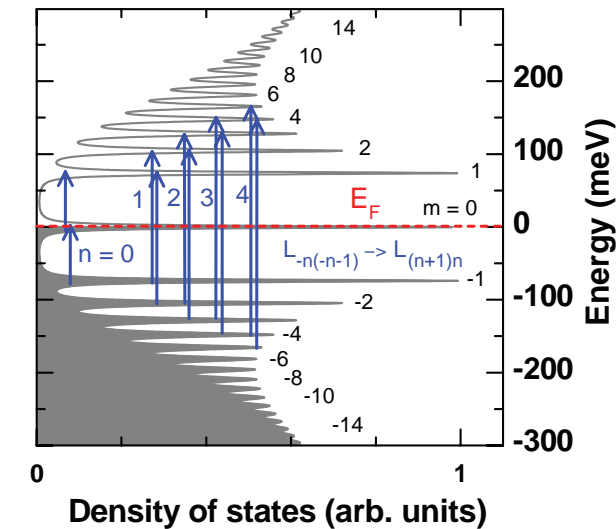
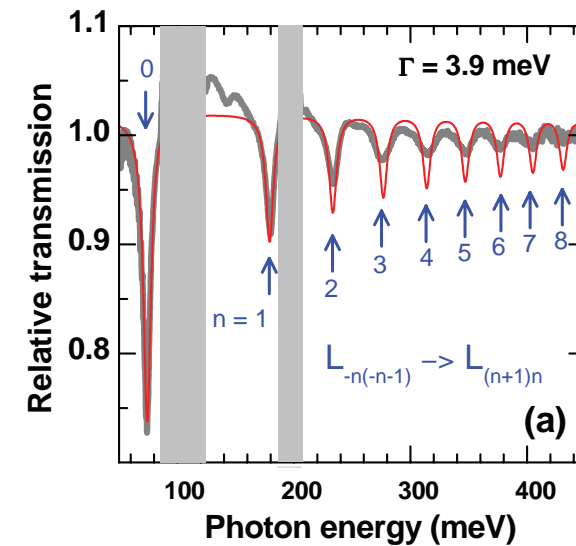
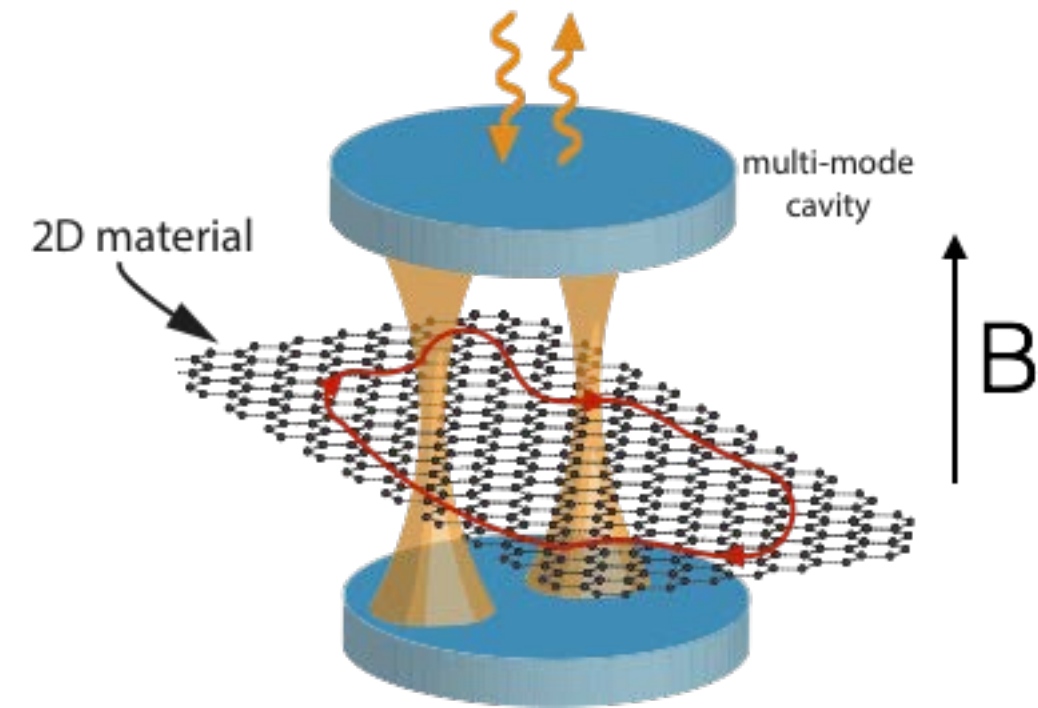
T. Grass (JQI)



A. Gazaryan (CUNY)

in collaboration with
Pouyan Ghaemi and A. Imamoglu

Use photons to detect/manipulate electronic topological states



Orlita, Potemski et al.
graphene $B = 4T$

$$H_{\text{quad}} \propto (p + eA)^2 \rightarrow \hbar\omega_c a^\dagger a$$

$$H_{\text{lin}} \propto \begin{pmatrix} 0 & v_f(p_x - ip_y + eA_x - ieA_y) \\ v_f(p_x + ip_y + eA_x + ieA_y) & 0 \end{pmatrix} \rightarrow \frac{v_f}{l_b} \begin{pmatrix} 0 & a \\ a^\dagger & 0 \end{pmatrix}$$

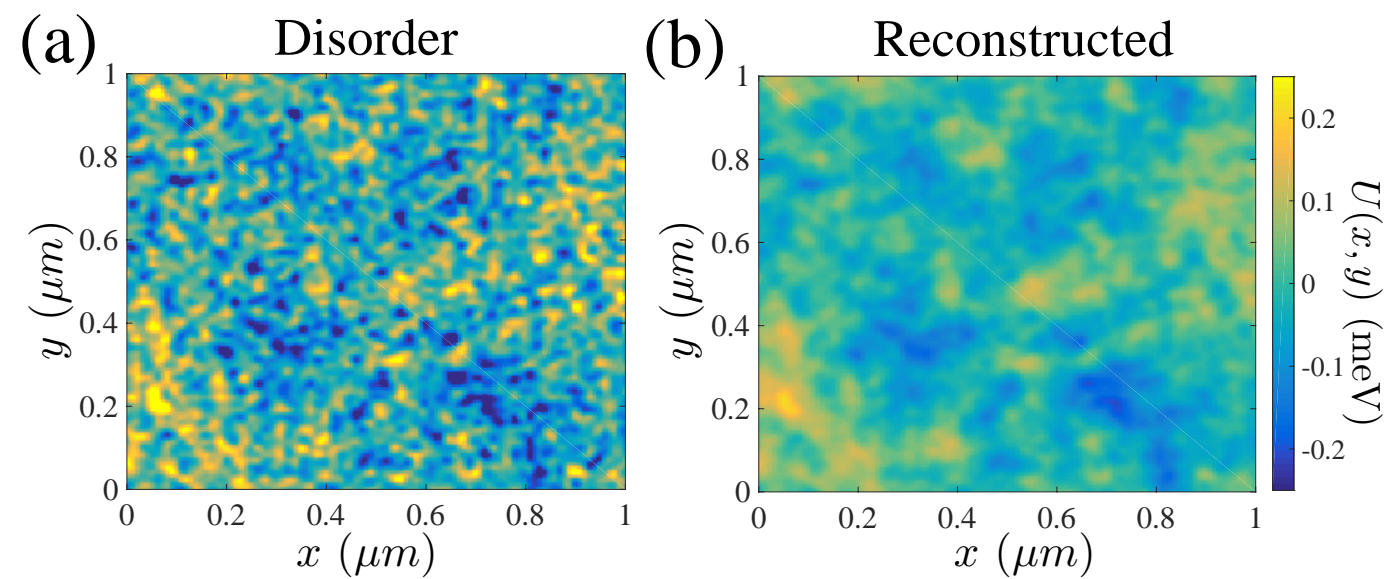
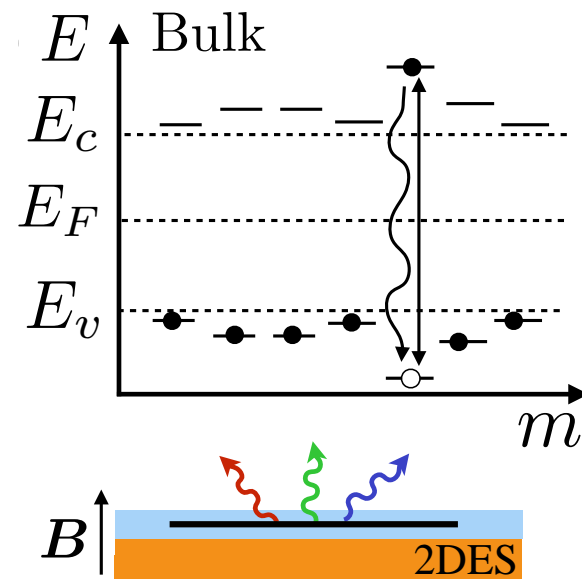
GaAs works by Pinczuk,....

Recently in cavity: Imamoglu Science (2016)

Dicke-superradiance: Ciuti, Fazio, MacDonald ..

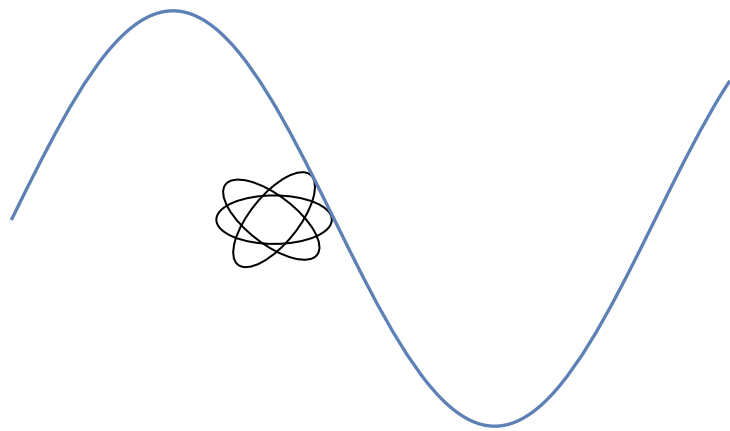
Mapping disorder landscape

$$H_{\text{dis}} = u_0(\mathbf{r})I + \mathbf{u}(\mathbf{r}) \cdot \boldsymbol{\tau}$$

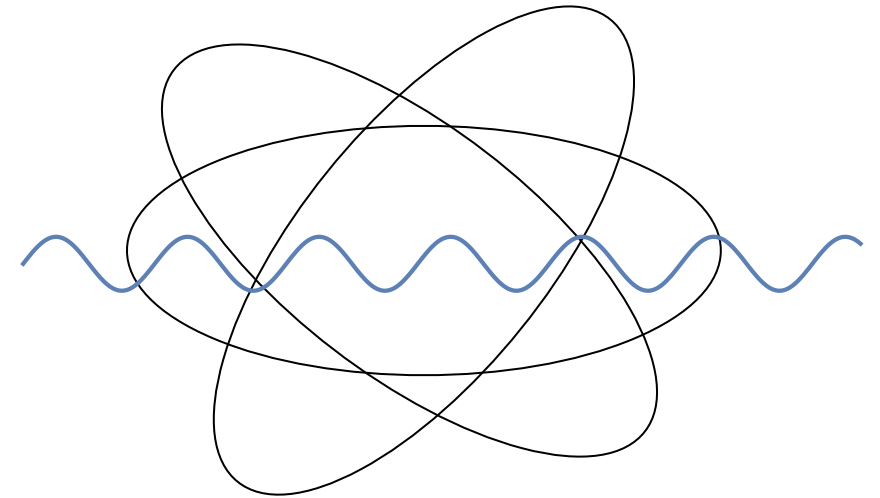


$$H_{\text{int}} = (-1)^s \frac{ev}{\sqrt{2}c} [\tau_+ A_+^*(x, y) + \tau_- A_-^*(x, y)] e^{-i\omega t} + h.c.$$

Most cases:
dipole approximation



Multipole emission



Extended states of electrons,
e.g. Quantum Hall, Rydberg excitations

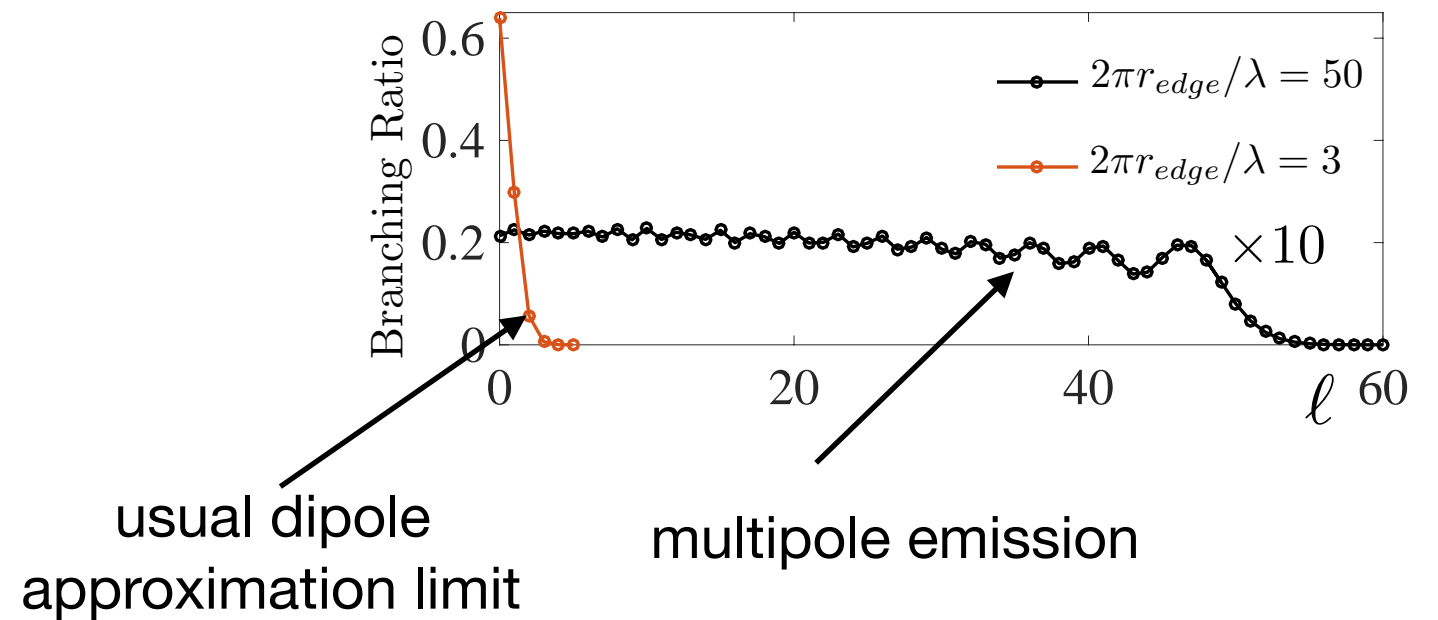
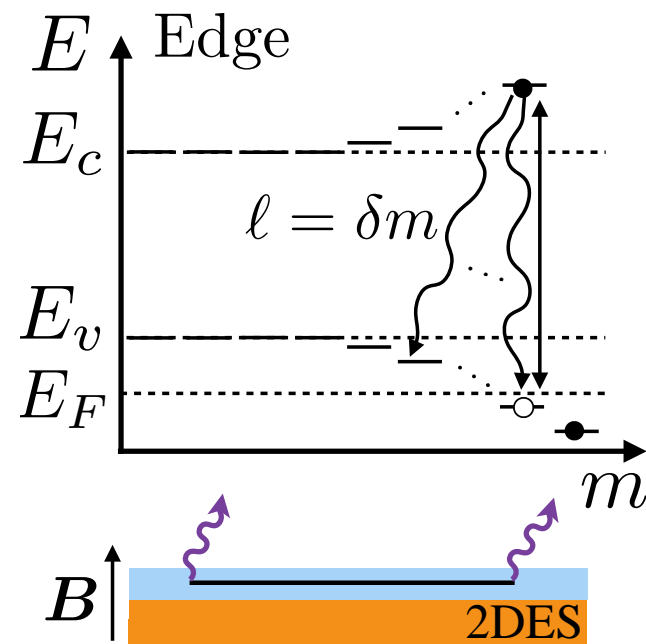
$$(re^{-i\theta})^{m'}$$

$$ev_f \langle 1, m' | \tau_+ \mathbf{A} | 0, m \rangle \rightarrow \delta_{m', m+l}$$

$$(re^{i\theta})^m$$

$$A \propto e^{ikz + il\theta} J_l(k_\perp r)$$

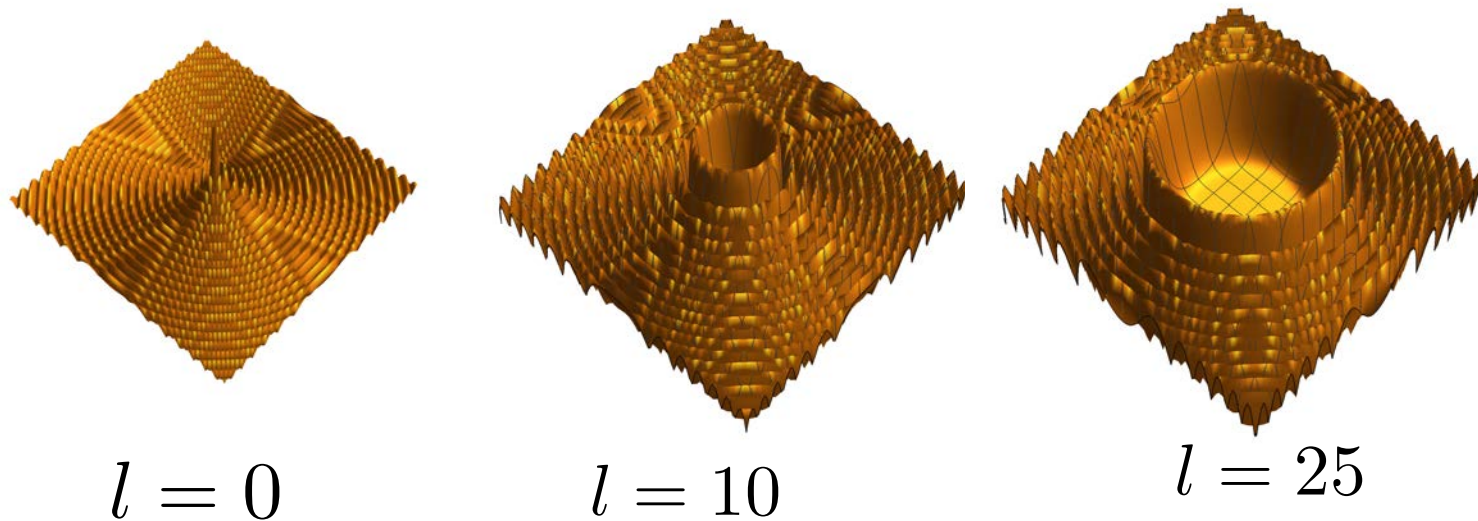
higher orbital angular momentum emission



Solutions of Maxwell's equations are given by Bessel functions:

$$k_{\perp} r_{vortex} = l$$

$$k_{\perp} = \sqrt{(\omega/c)^2 - k_{\parallel}^2}$$



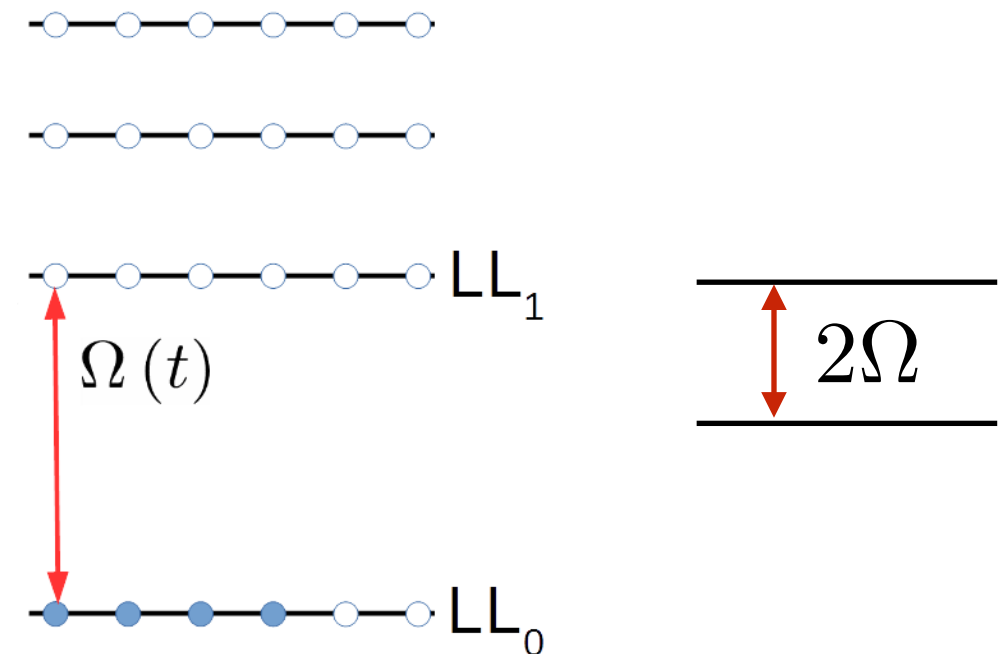
so the maximum OAM:

$$\frac{2\pi}{\lambda} r_{edge} = l_{max}$$

Synthetic bilayer Graphene (?)

Use light to couple two LLs:

- (1) Different LL plays the role of layers
- (2) Light plays the role of tunneling



- What type of states can one engineer with light?

$$\mathcal{H}_{\text{int}} = \sum_{\{n,j\}} A_{n_3,j_3,n_4,j_4}^{n_1,j_1,n_2,j_2} \delta_{n_1+n_2,n_3+n_4} c_{n_1,j_1}^\dagger c_{n_2,j_2}^\dagger c_{n_3,j_3} c_{n_4,j_4}$$

Haldane pseudo potential for synthetic bilayer

$$\hat{V} = \sum_M \left[\sum_{m \text{ odd}} (V_m^\uparrow |mM, \uparrow\uparrow\rangle \langle mM, \uparrow\uparrow| + V_m^\downarrow |mM, \downarrow\downarrow\rangle \langle mM, \downarrow\downarrow|) + \right. \\ \sum_m V_m^\parallel (|mM, \uparrow\downarrow\rangle \langle mM, \uparrow\downarrow| + |mM, \downarrow\uparrow\rangle \langle mM, \downarrow\uparrow|) + \\ \left. \sum_m V_m^\times (|mM, \uparrow\downarrow\rangle \langle mM, \downarrow\uparrow| + |mM, \downarrow\uparrow\rangle \langle mM, \uparrow\downarrow|) \right]$$

intra-layer: breaks Z_2
(not true in conventional bilayer)

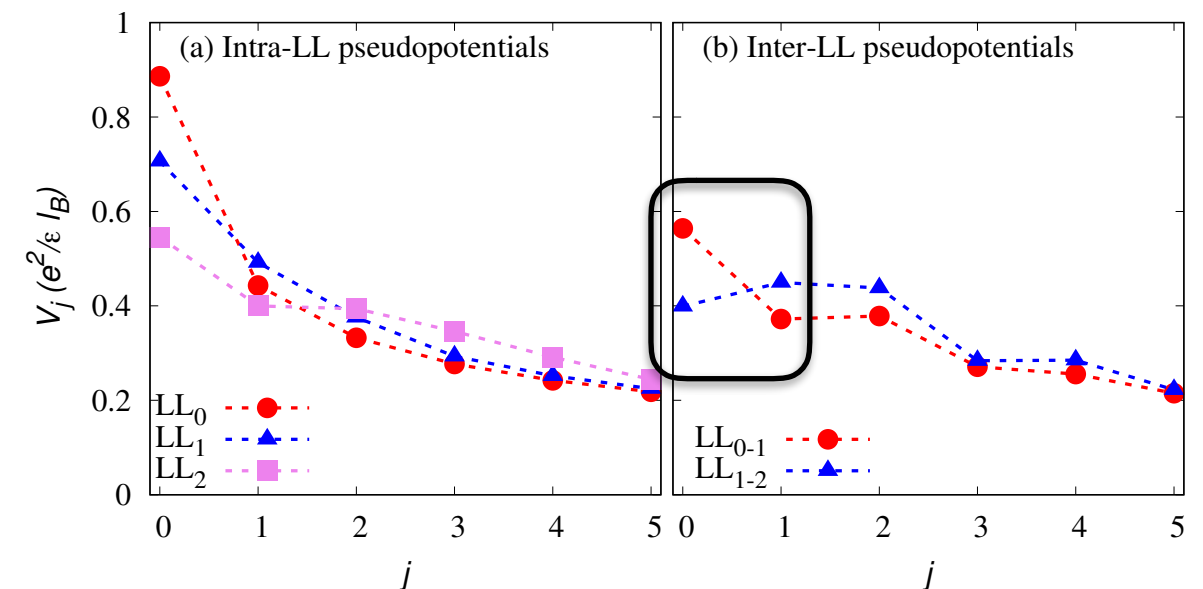
inter-layer: direct

inter-layer: cross
(absent in conventional bilayer)

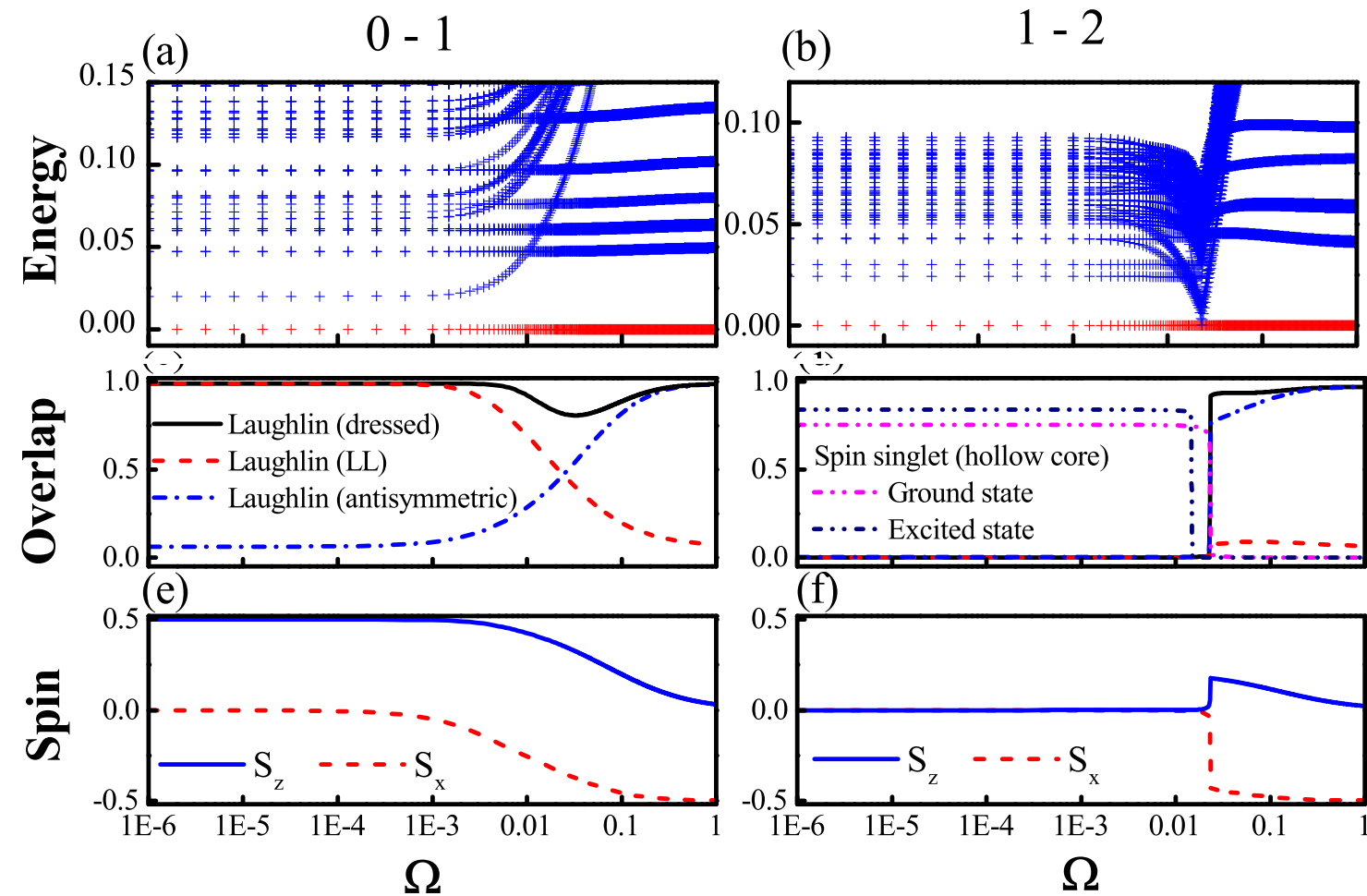
$$|+\rangle = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle) \\ |-\rangle = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$

$$\hat{V} = \sum_M \left[\sum_{m \text{ odd}} V_m^\uparrow (|mM, \uparrow\uparrow\rangle \langle mM, \uparrow\uparrow| + V_m^\downarrow |mM, \downarrow\downarrow\rangle \langle mM, \downarrow\downarrow|) + \right. \\ \left. \sum_{m \text{ odd}} [V_m^\parallel + V_m^\times] |mM, +\rangle \langle mM, +| + \sum_{m \text{ even}} [V_m^\parallel - V_m^\times] |mM, -\rangle \langle mM, -| \right]$$

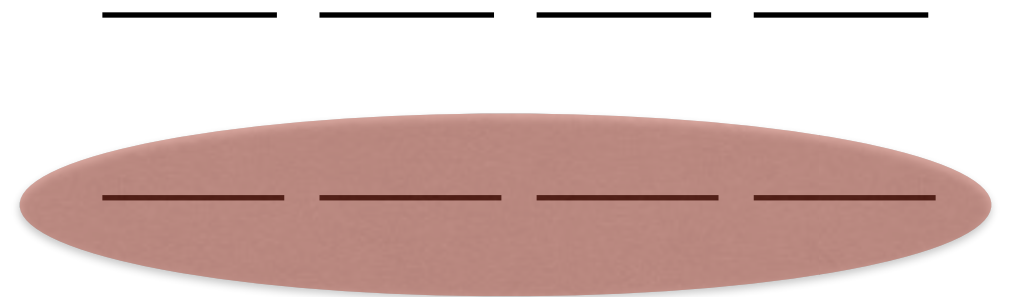
$$V_m^{\text{inter}} = \begin{cases} V_m^\parallel + V_m^\times & \text{if } m \text{ is odd,} \\ V_m^\parallel - V_m^\times & \text{if } m \text{ is even} \end{cases}$$



- Filling factor is $\nu = 2/3$



Dressed Laughlin $\Omega \gg e^2/l_B$



$$z_i^m \longrightarrow \prod_{i < j} (z_i - z_j)^m$$

	Sphere	Disk	Torus
$\nu = 1/2$	0.85 ($N = 6$)	0.97	0.83 ($\mathbf{K} = \mathbf{0}$)
(HR)	0.75 ($N = 8$)	$(N = 6, L = 24)$	0.72 ($\mathbf{K} \neq \mathbf{0}$)
	0.72 ($N = 10$)		$(N = 8)$
$\nu = 2/3$	0.99 ($N = 4$)	0.81 ($N = 6, L = 18$)	
(IP)	0.55 ($N = 8$)	0.63 ($N = 8, L = 36$)	
	0.39 ($N = 12$)		

For bilayer: McDonald Haldane PRB (1996)
Recently: Peterson, Barkeshli, Wen, Vaezi, ...

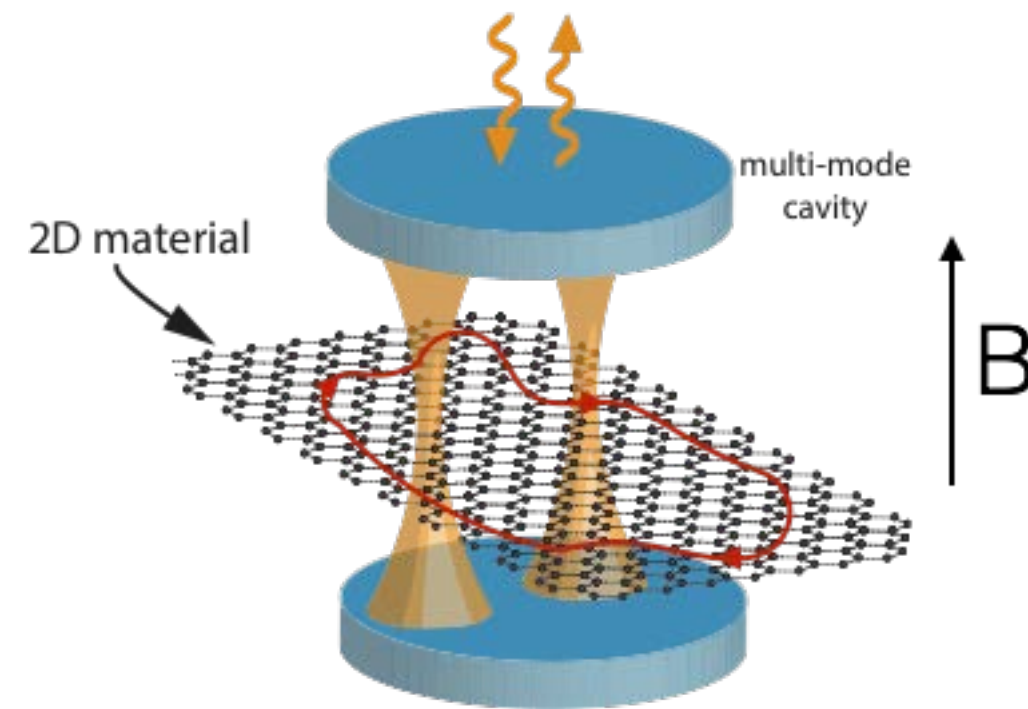
Outlook:

Thermalization in the driven system: Can phonons cool the system in the rotating frame?

Dehghani, Oka, and Mitra, PRB(2014)

Iadecola and Chamon PRB (2015)

Seetharam, Bardyn, Lindner, Rudner, and Refael, PRX (2015)

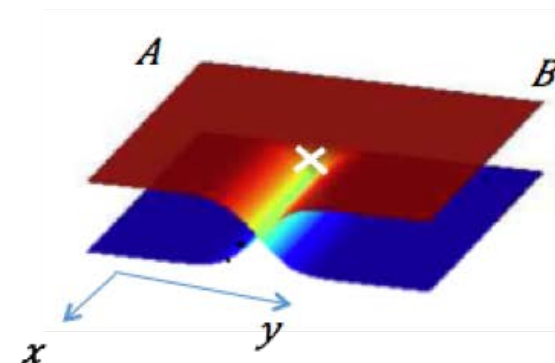


Engineering tunneling, interaction

Can we optically create and manipulate anyons?

Constructing twist defects?

Barkeshli, Qi PRX (2014)



- Vikram Orre
- Hwanmun Kim
- Bin Cao
- Chris Flower
- Alireza Seif
- Zepei Can
- Sabyasachi Barik
- Julia Sell
- Jon Vannucci
- Tobias Huber
- Wade DeGottardi
- Tobias Grass
- Guanyu Zhu
- Sunil Mittal

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- P. Ghaemi, A. Ghazaryan (CUNY)
- A. Vaezi (Stanford)
- R. Salem (PicoLuz/Thorlabs)
- A. Imamoglu (ETHZ)

graduate and
postdoc
fellowships

[publications: hafezi.umd.edu](http://publications:hafezi.umd.edu)

