Topological effects and quantum Hall physics with light

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Textbook

Topological Photonics

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(Dated: February 13, 2018)

Topological photonics is a rapidly-emerging field of research in which geometrical and topological ideas are exploited to design and control the behavior of light. Drawing inspiration from the discovery of the quantum Hall effects and topological insulators in condensed matter, recent advances have shown how to engineer analogous effects also for photons, leading to remarkable phenomena such as the robust unidirectional propagation of light, which hold great promise for applications. Thanks to the flexibility and diversity of photonics systems, this field is also opening up new opportunities to realise exotic topological models and to probe and exploit topological effects in new ways. In this article, we review experimental and theoretical developments in topological photonics across a wide-range of experimental platforms, including photonic crystals, waveguides, metamaterials, cavities, optomechanics, silicon photonics and circuit-QED. We discuss how changing the dimensionality and symmetries of photonics systems has allowed for the realization of different topological phases, and we review progress in understanding the interplay of topology with non-Hermitian effects, such as dissipation. As an exciting perspective, topological photonics can be combined with optical nonlinearities, leading towards new collective phenomena and novel strongly-correlated states of light, such as an analogue of the fractional quantum Hall effect.

arXiv:1802.04173v1 [physics.optics] 12 Feb 2018

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To (hopefully) appear soon on RMP...

Standing on the shoulders of giants

Klaus von Klitzing
Prize share: 1/1

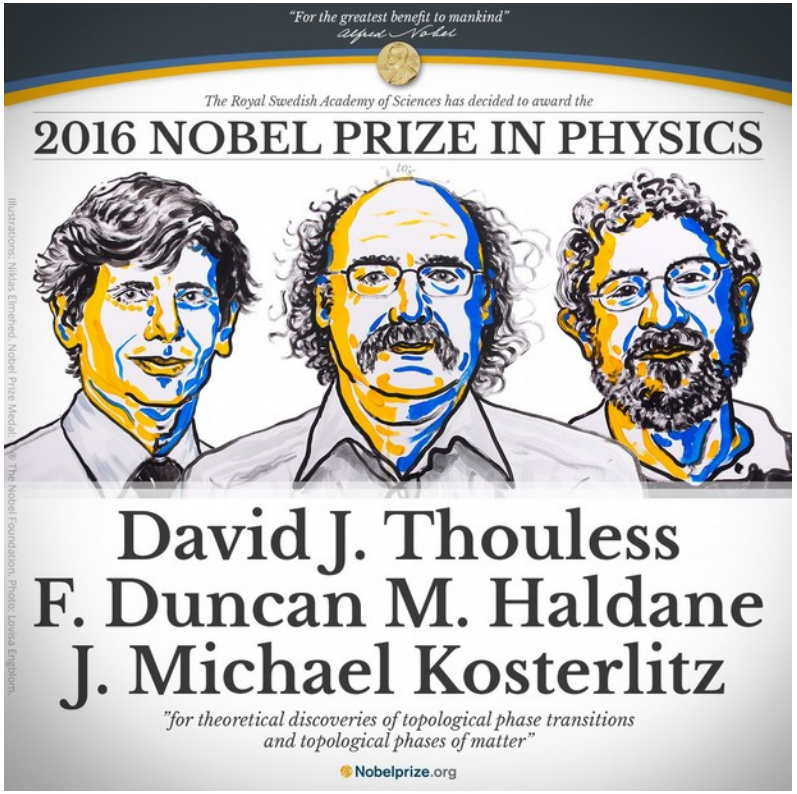
The Nobel Prize in Physics 1985 was awarded to Klaus von Klitzing *"for the discovery of the quantized Hall effect"*.

Robert B. Laughlin
Prize share: 1/3

Horst L. Störmer
Prize share: 1/3

Daniel C. Tsui
Prize share: 1/3

The Nobel Prize in Physics 1998 was awarded jointly to Robert B. Laughlin, Horst L. Störmer and Daniel C. Tsui *"for their discovery of a new form of quantum fluid with fractionally charged excitations"*.



At all events, as often as tidings were brought that Philip had either taken a famous city or been victorious in some celebrated battle, Alexander was not very glad to hear them, but would say to his comrades: 'Boys, my father will anticipate everything; and for me he will leave no great or brilliant achievement to be displayed to the world with your aid. (Plutarch, *Life of Alexander* – Chap. 5.2)

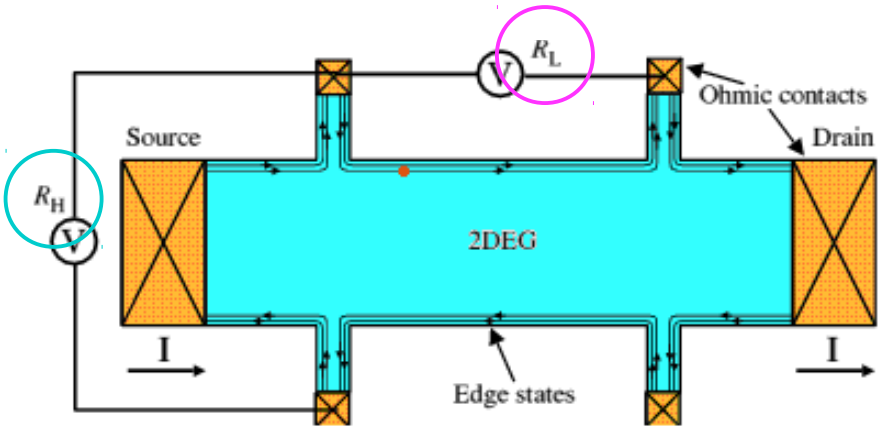
→ As history demonstrated, many more discoveries waiting for you!!

Integer & Fractional Quantum Hall effect

Thin and extremely clean 2D electron gas
(standard object in solid state physics)

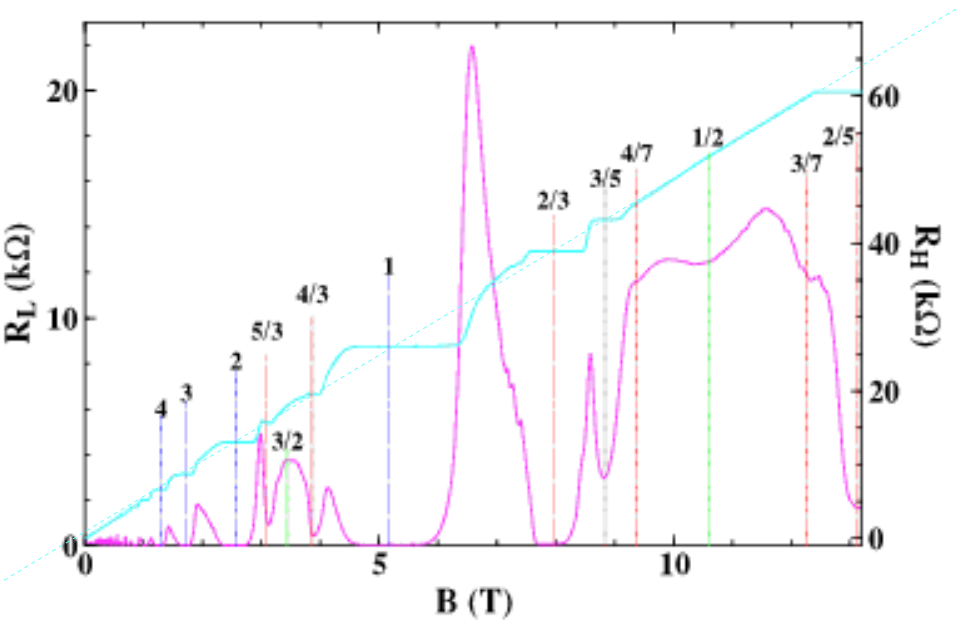
Measure longitudinal and
transverse resisitivity

Textbook Hall effect: $R_H \sim B$



Very low T → features @ rational $1/\nu = B/B_0$

- R_L drops to zero
- R_H shows plateaux

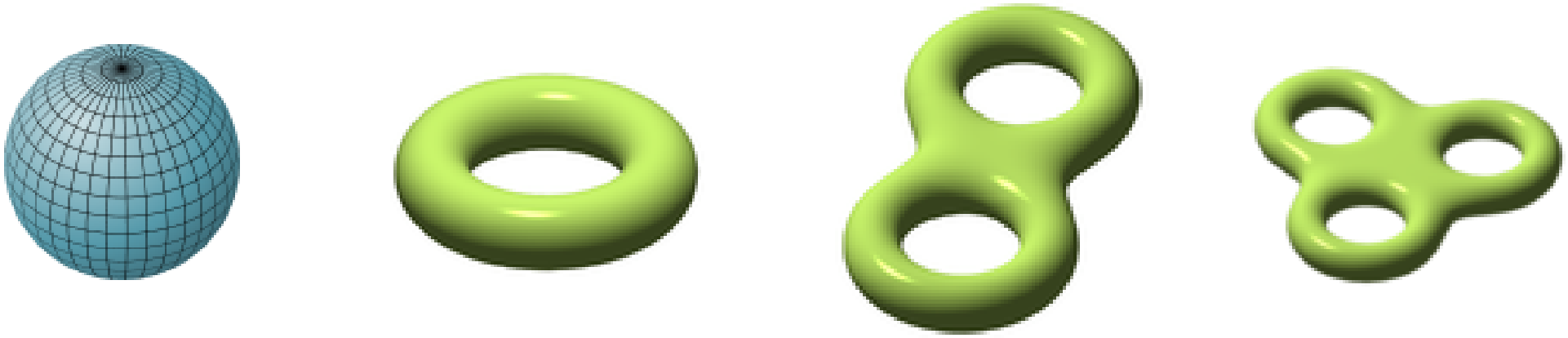


Effect rather insensitive to disorder
Disorder sets extension of plateaux

How to explain it?

Nobel prizes: Von Klitzing (1985); Laughlin, Stoermer, Tsui (1998)

What is (global) topology?



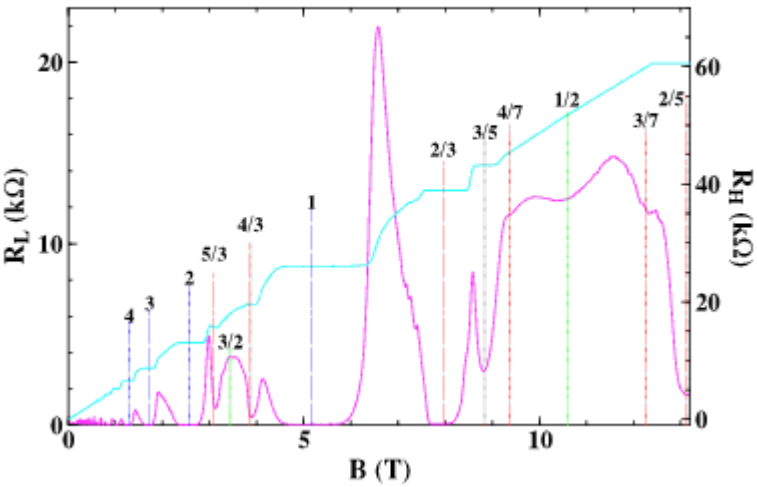
Sphere, torus, etc...

- Distinction robust against continuous deformations
- Need to tear surface to transform into each other
- Not affected if surface is (a bit) rough

Quantified by topological invariant g:

- genus g = number of holes
- related to Euler characteristic $\chi = V - E + F = 2 - 2g$
- and to integral of Gaussian curvature

$$\int_S d^2 r \; \kappa(r) = 2\pi\chi = 4\pi(1-g)$$



Quantization of R_H to rational values
Rather insensitive to disorder

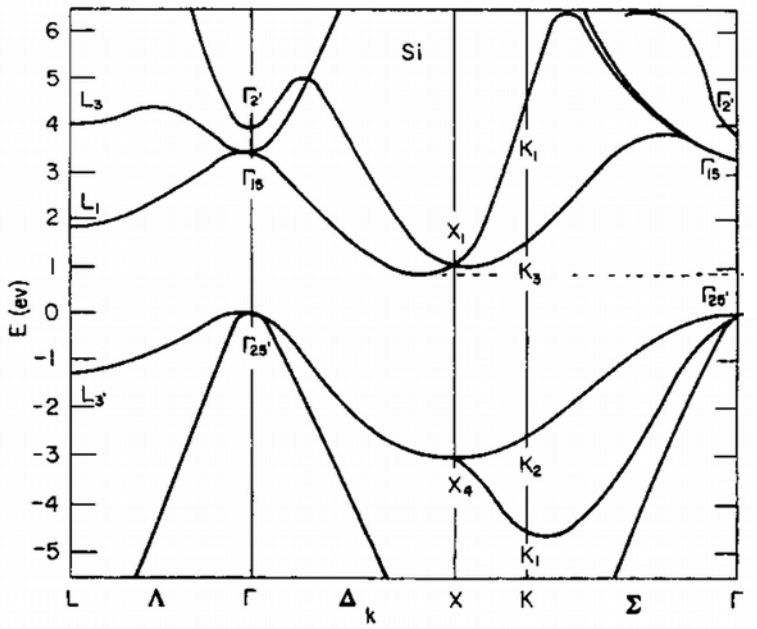
Electron band theory of solids

High-school solid-state physics/chemistry:

- electrons in solids occupy delocalized quantum orbitals
- Organized in energy bands E(k) with k in FBZ of the lattice

- all states upto a gap occupied → insulator
- if band partially filled → metal

Band structure of Silicon



Same concept of bands applies to sound waves in periodic materials or metamaterials, to optical waves in photonic crystals, to matter waves in optical lattices, etc.

But surprises are often just beyond the corner...

Electron states in solids → another (geometrical) property:

- Defines a Berry connection: $A_{n,k} = i\langle u_{n,k} | \nabla_k u_{n,k} \rangle$
- And its Berry curvature: $\Omega_{n,k} = \nabla_k \times A_{n,k}$

Connections on compact manifolds (e.g. toroidally-shaped FBZ)

→ integer-valued topological invariants, e.g. Chern number $\int_{FBZ} \Omega_{n,k} d^2k = 2\pi C_n$

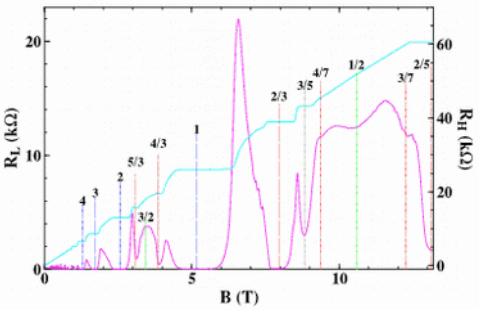
Celebrated TKNN paper (Thouless, Kohmoto, Nightingale, Den Nijs, PRL '82):

- Filled band → no longitudinal conductance
- Transverse Hall conductivity ↔ Chern # of occupied bands $j^y = \frac{E_x}{(2\pi)^2} \int_{T^2} \Omega_{n,k} d^2k = \frac{\nu_1}{2\pi} E_x$

Explains quantized transverse conductance and vanishing longitudinal at integer B/B_0

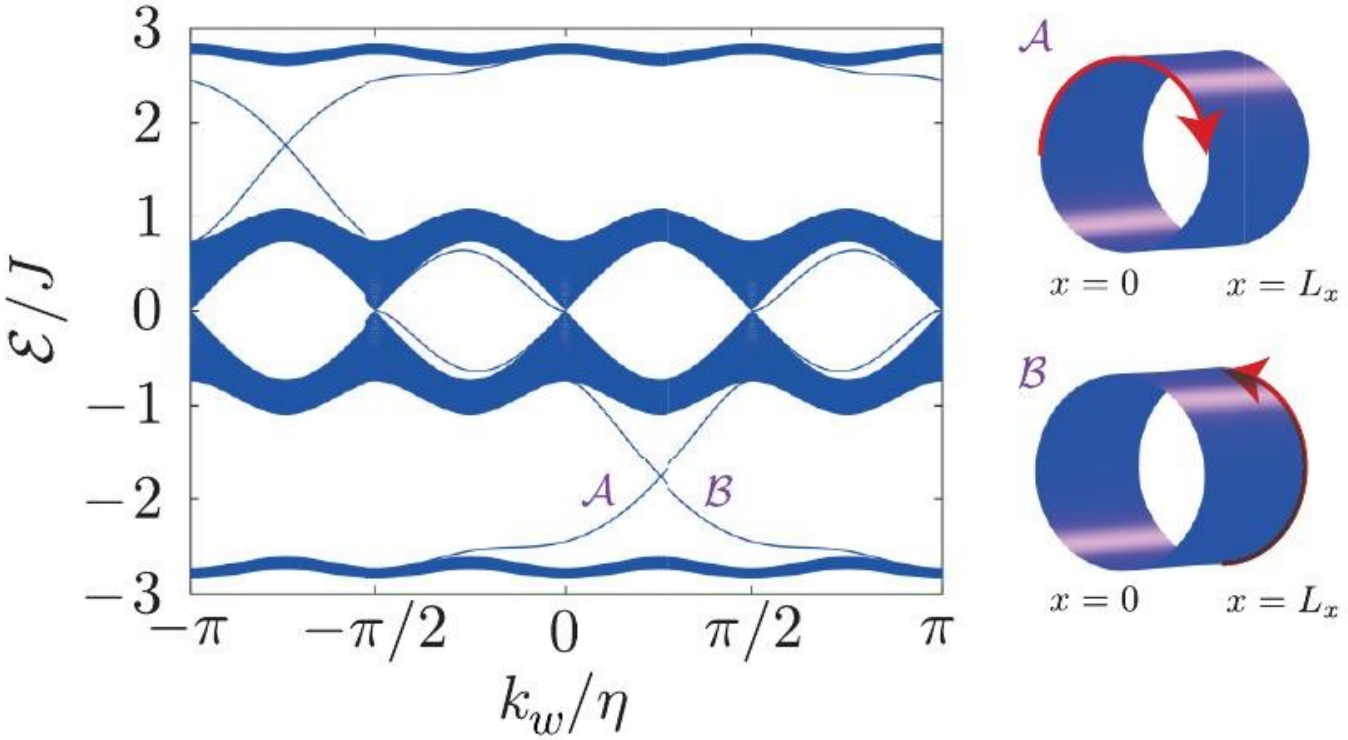
Effect of disorder:

- localized states, not contributing to conduction
- Explains finite-width plateaux



Electron gas in an (integer) QH state is prime example of "topological (Chern) insulator"

Bulk-boundary correspondance



Whenever interface between media with different Chern number:

- Unidirectionally propagating edge modes
- Number and chirality determined by Chern number difference
- Robust to disorder

Alternative way of seeing quantized conductance in quantum Hall effect

These lectures in a nutshell:

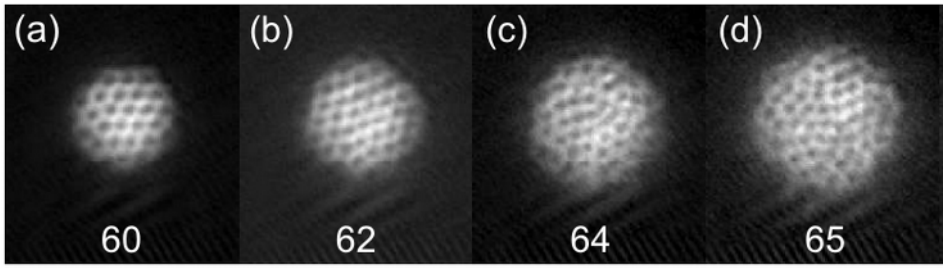
- *How to make neutral particles such as photons to feel a Lorentz force?*
- *Can this be used to study topological effects ?*
- *What about integer/fractional quantum Hall states ?*
- *Can one expect new many-body physics?*

Part 1:

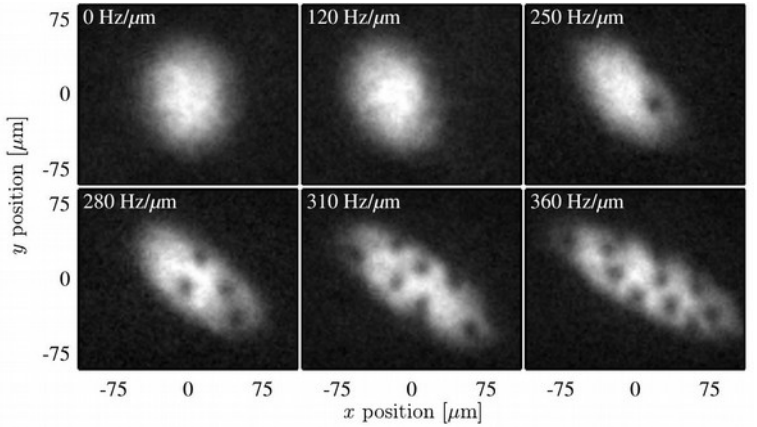
Magnetism with light and topological photonics

How to make (neutral photons) to feel a Lorentz force

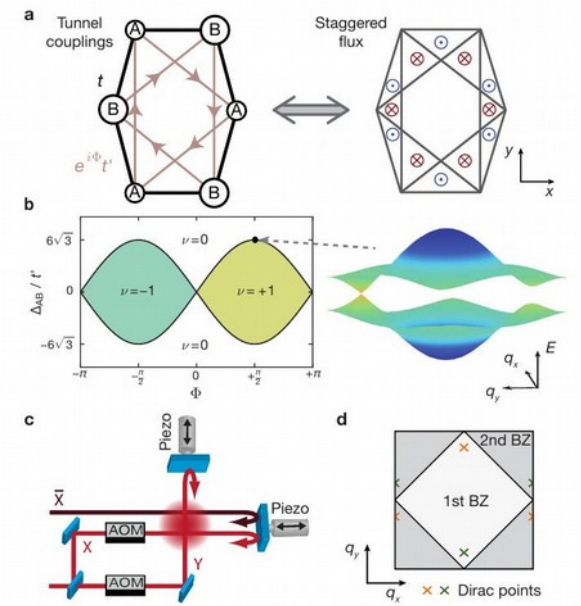
Synthetic gauge fields for atoms *et similia*



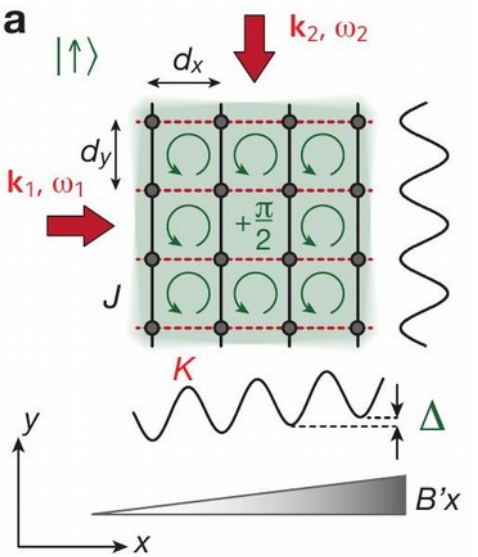
Rotating frame: Coriolis mathem. equivalent to Lorentz
→ generates vortices in atomic gas
Bretin et al., PRL 92, 050403 (2004)



Raman processes + (physical) magnetic field
→ synthetic magnetic field
Lin et al. Nature 471, 83 (2011)



Periodically shaken lattice
→ Haldane model for atoms
Jotzu et al., Nature 515, 237-240 (2014)

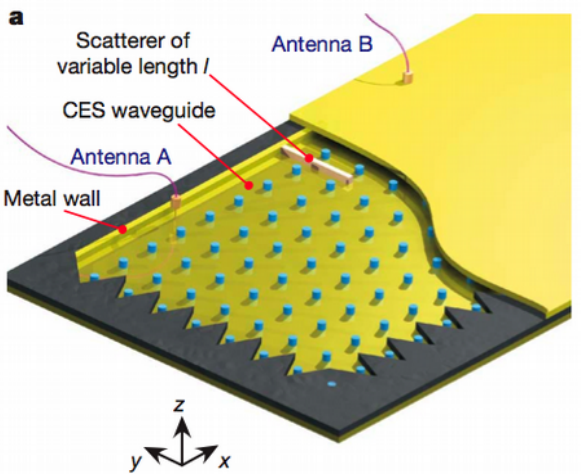


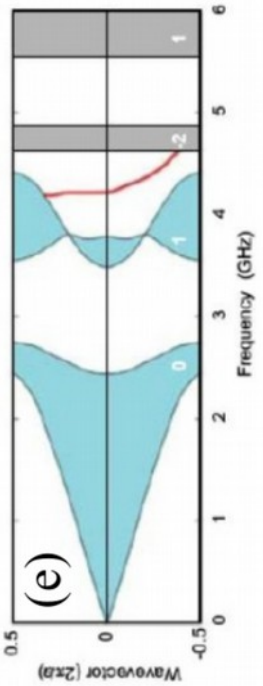
Photon-assisted tunneling
→ 2D Harper-Hofstadter model
Aidelsburger et al., PRL 111, 185301 (2013)

2009 - Photonic (Chern) topological insulator

MIT '09, Soljacic group

Original proposal Haldane-Raghu, PRL 2008





Magneto-optical photonic crystals for μ-waves

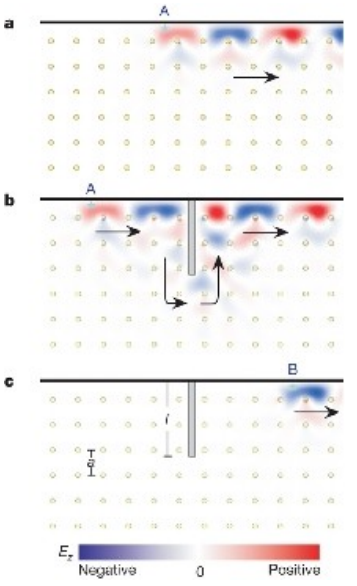
T-reversal broken by magnetic elements

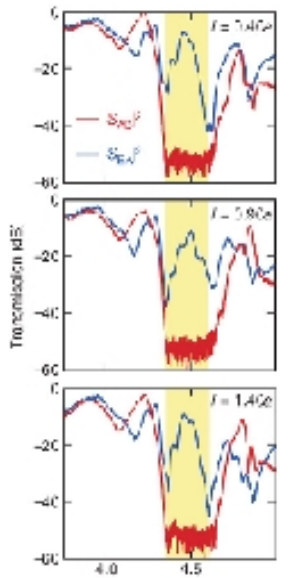
Band wih non-trivial Chern number:

→ chiral edge states within gaps

Experiment:

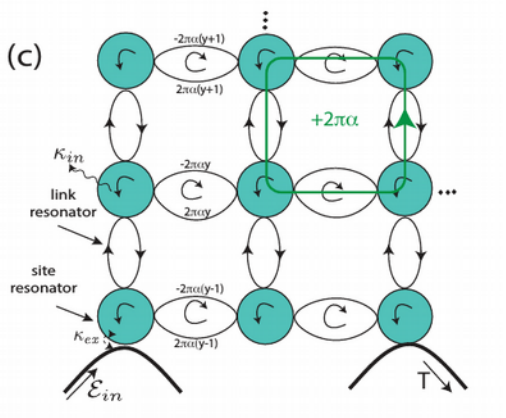
- measure transmission
  from antenna to receiver
- only in one direction:
  unidirectional propagation
- immune to back-scattering by defects
  topologically protected

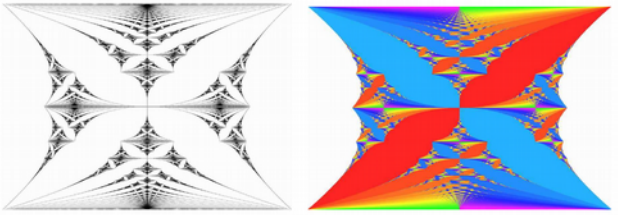




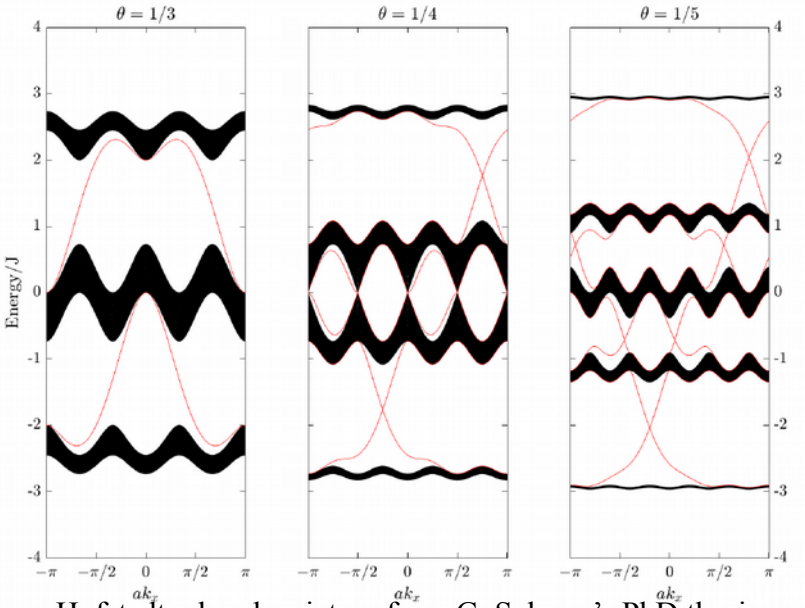
Wang et al., Nature 461, 772 (2009)

Harper-Hofstadter model





Avron et al's colored Hofstadter butterfly
color=Chern # of gap



Hofstadter bands, picture from G. Salerno's PhD thesis

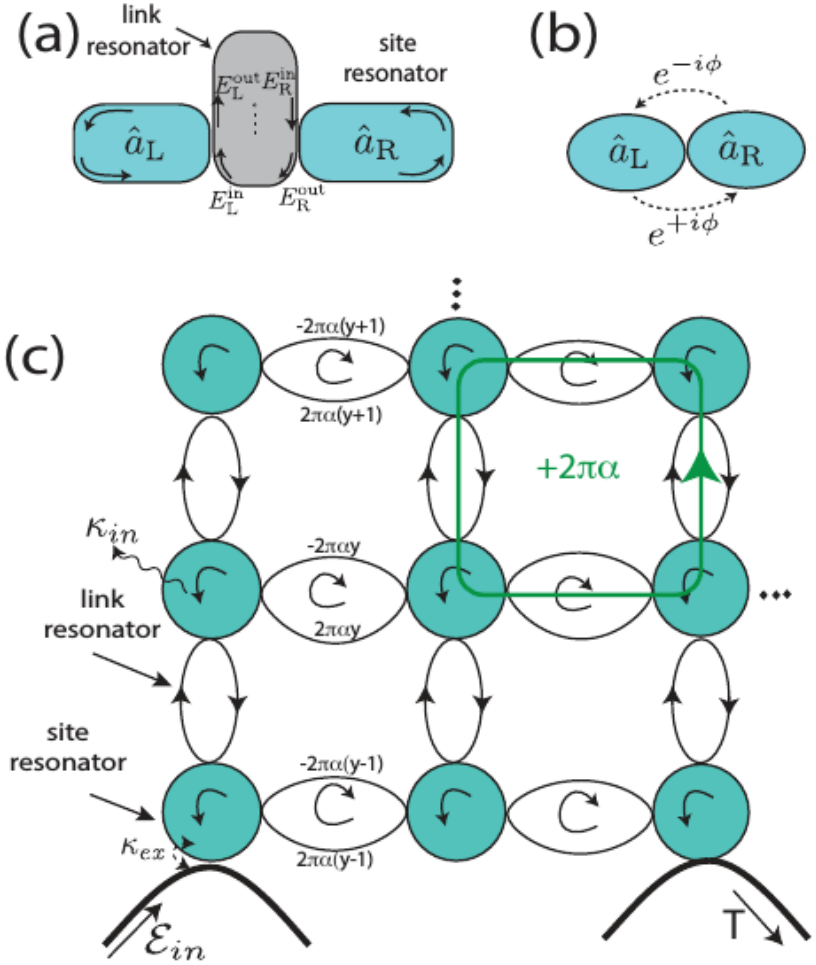
2D square lattice at large magnetic flux

- eigenstates organize in bulk Hofstadter bands
- Berry connection in k-space: $A_{n,k} = i\langle u_{n,k} | \nabla_k u_{n,k} \rangle$
- Berry curvature: $\Omega_{n,k} = \nabla_k \times A_{n,k}$
- Integer-valued topological invariant: Chern number $\int_{FBZ} \Omega_{n,k} d^2k = 2\pi C_n$

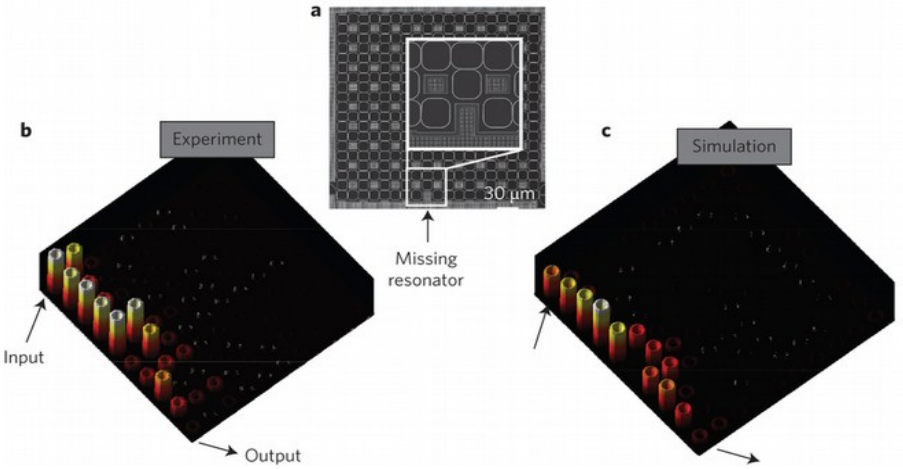
If Chern number is non-zero:

- chiral edge states within gaps (bulk-edge correspondence)
- unidirectional propagation
- (almost) immune to scattering by defects

2013 - Photonic realization of HH model



- Array of CMOS Silicon ring cavities (whispering gallery modes)
- Coupled via off-resonant ancilla cavities
- Different optical path in either direction
  → non-trivial hopping phase
- Light injected from input waveguide and detected from output and from upwards scattering
- Signature of topology → chiral edge states
- But… what about the reciprocity theorem?



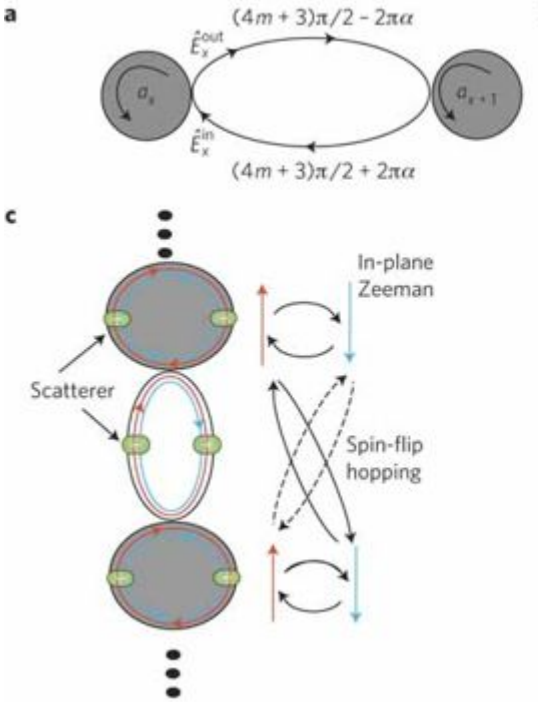
Hafezi et al., Nat. Phot. **7**, 1001 (2013) → new developments in Mohammad's seminar this afternoon !!

Quantum Hall vs. Spin Hall

Each cavity supports two degenerate CW and CCW modes

→ spin-1/2 degree of freedom

- Opposite hopping phase for two spin states
- Opposite chirality of edge states
- Topological protection ensured by weakness of scattering



In electronic topological insulators:

- different wave equation, different coupling to perturbations
- spin-flips forbidden by T-reversal (Kramers degeneracy)
- spin-flips allowed for e.m. wave equation, in a guru's word: $T^2=-1$ instead of $T^2=1$
- spin-orbit coupling preserves T-reversal
- topological invariant $\mathbf{Z}_2$ instead of $\mathbf{Z}$

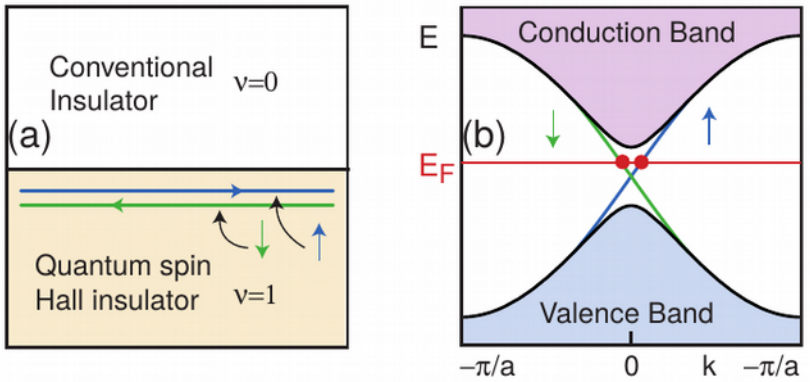
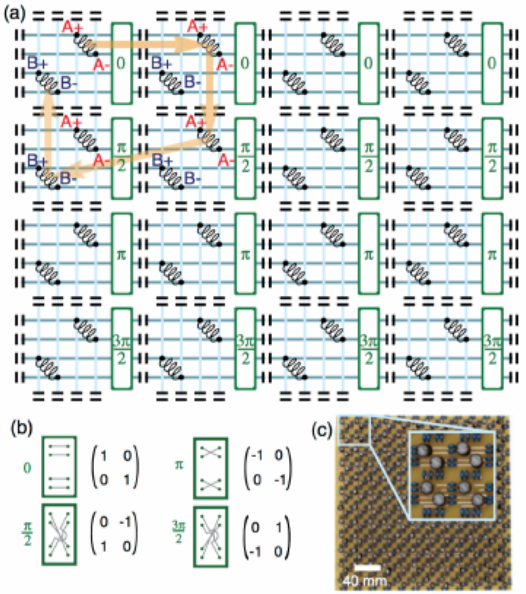
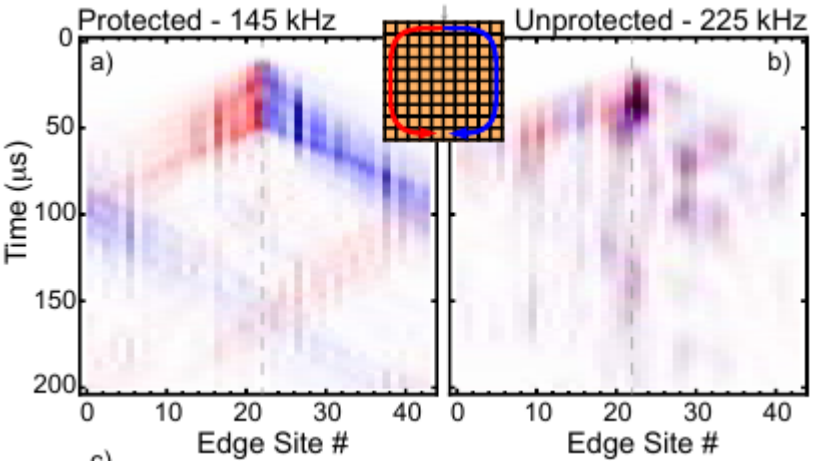


Figure from Hasan-Kane, RMP 2011
An intro in Kane-Moore, Phys. World 2011

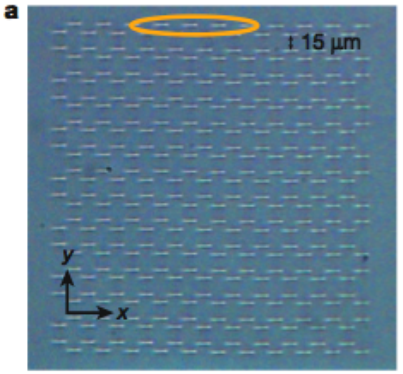
2013 & ff - Other topological models in photonics

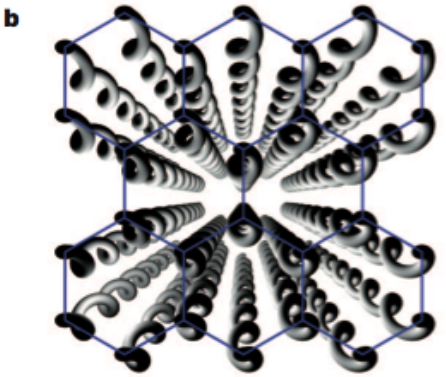
Lumped-element circuits
Ningyuan et al., PRX (2015)

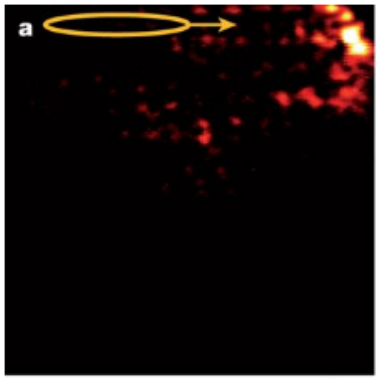




Fusilli-shaped waveguides in glass:
propagating geometry,
Floquet bands
Rechtsman, et al., Nature (2013)

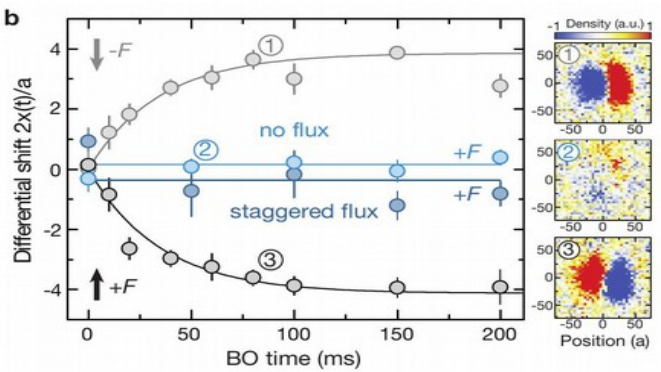




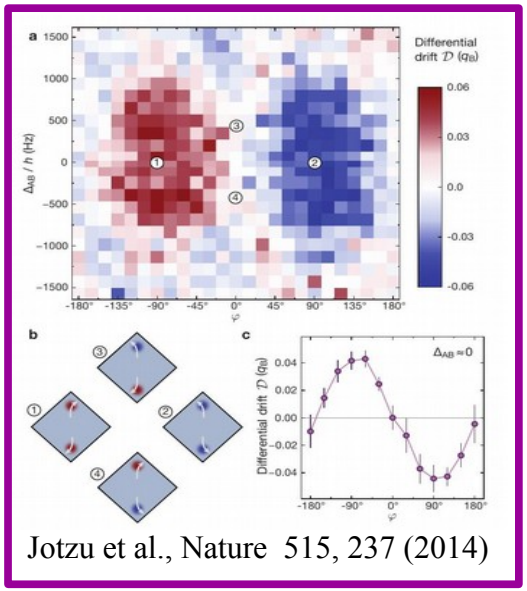


… and many others! See long review Ozawa et al., 1802.04173

How to observe geometrical & topological properties of *bulk* ?



Aidelsburger et al. Nat. Phys. 11, 162 (2015)



Jotzu et al., Nature 515, 237 (2014)

Experiments with atoms

Prototype effect → Integer Quantum Hall

- Requires filled band
- Quantized transverse current $j^y = \frac{E_x}{(2\pi)^2}\int_{T^2}\Omega_{n,k}\,d^2k = \frac{\nu_1}{2\pi}E_x$

How to observe it with light?

Semiclass. EoM:

$$\hbar\dot{\mathbf{k}}_c(t) = e\mathbf{E}\,,$$
$$\hbar\dot{\mathbf{r}}_c(t) = \nabla_{\mathbf{k}}\mathcal{E}_{n,\mathbf{k}} - e\mathbf{E}\times\boldsymbol{\Omega}_n(\mathbf{k})$$

Berry curvature → sort of **k**-space magnetic field

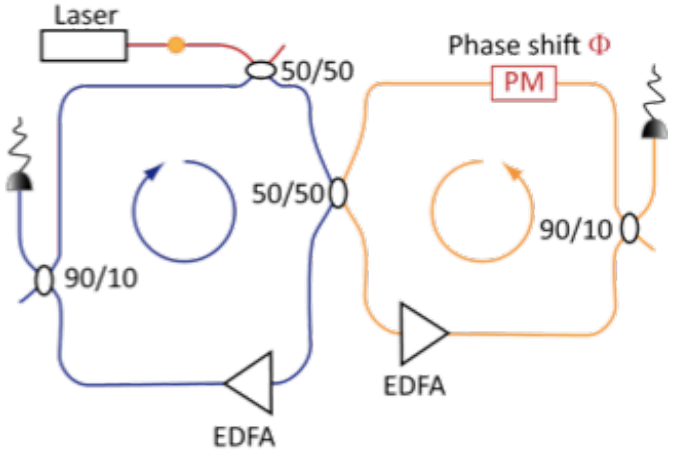
Lateral displacement analogous to Lorentz force

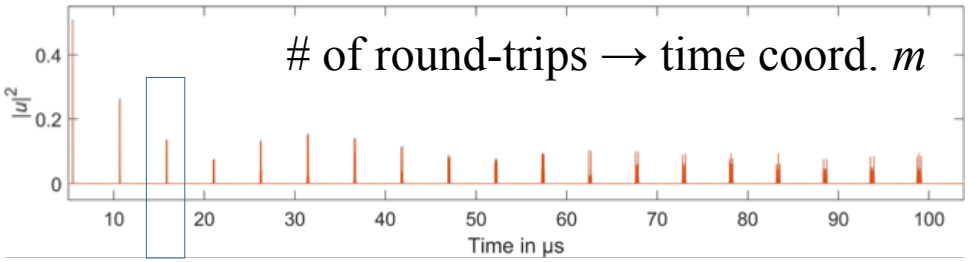
Depending on band filling:

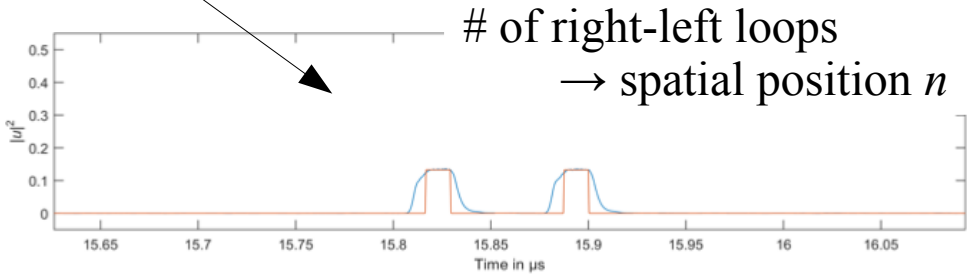
Anomalous vs. Integer Quantum Hall effect

An old concept, see e.g. review in Xiao-Chang-Niu, RMP 82, 1959 (2010).
First proposals for atoms: Dudarev, IC et al. PRL 92, 153005 (2004)
Price-Cooper, PRA 83, 033620 (2012)

2016 - Experimental mapping of Berry curvature

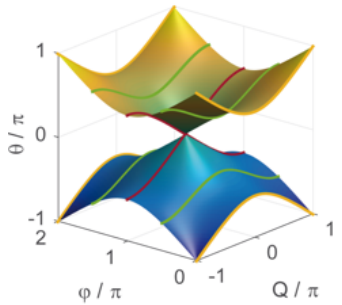






Periodic temporal modulation of $\Phi(m)=\pm\varphi$:

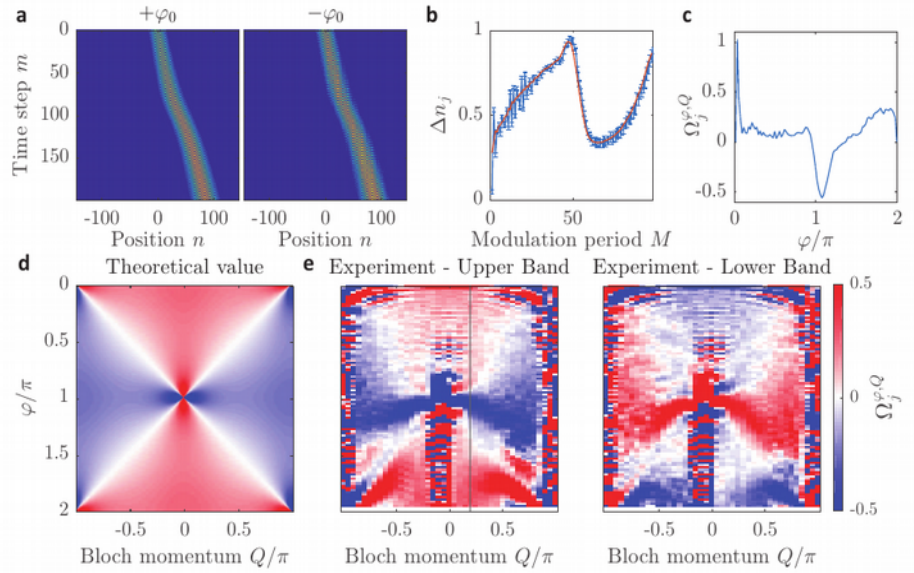
- 1D Floquet band structure $\theta(Q,\varphi)$, φ considered as 2nd dim



- Berry curvature

$$\Omega_j^{\varphi,Q} = \frac{\partial}{\partial\varphi}\langle\psi_j|i\frac{\partial}{\partial Q}|\psi_j\rangle - \frac{\partial}{\partial Q}\langle\psi_j|i\frac{\partial}{\partial\varphi}|\psi_j\rangle$$

- Geometrical charge pumping if φ adiabatically varied
- Look at lateral displacement along n at all times m → reconstruct Berry curvature $\Omega_j^{(\varphi,Q)}$ in whole FBZ



Cold atoms → Berry phase reconstructed via state tomography (Fläschner et al., Science '16)

Wimmer, Price, IC, Peschel, Nat. Phys. 2017

Driven-dissipative photonic system

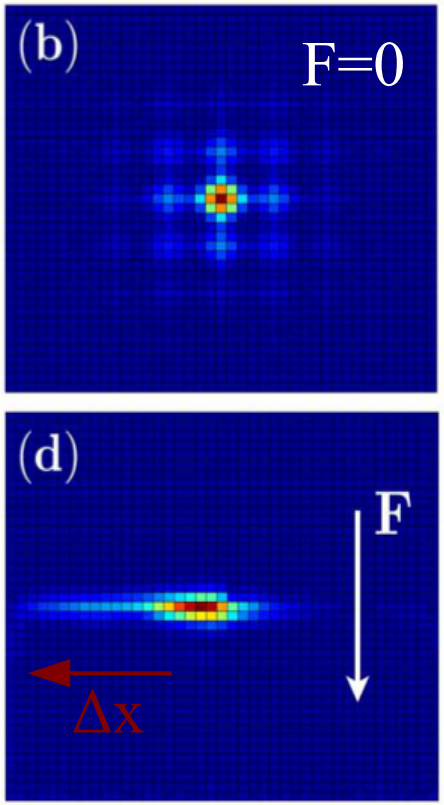
Cavity lattice geometry → promising in view of interacting photon gases, but radiative losses.

Short time to observe BO's, but experiment @ non-eq steady state even better

Coherent pumping $H_d = \sum_i F_i(t)\,\hat{b}_i + F_i^*(t)\,\hat{b}_i^\dagger$ + losses at rate γ

Pump spatially localized on central site only:

- couples to all k's within Brillouin zone
- resonance condition selects specific states
- can be used to control band filling



In the presence of force F:

motion in BZ → lateral drift in real space by Berry curvature

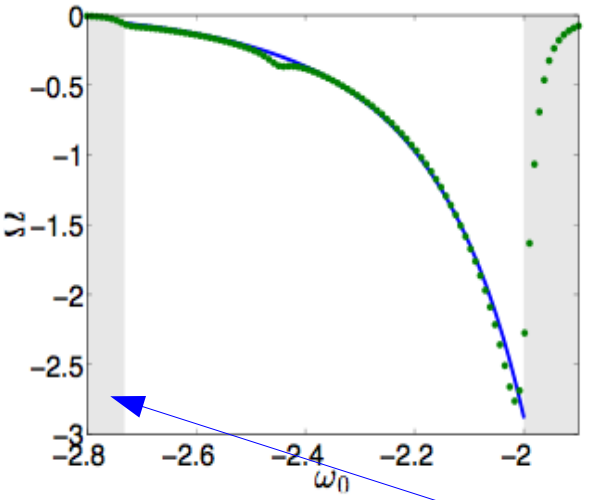
$$\hbar\dot{\mathbf{k}}_c(t) = e\mathbf{E}\,,$$
$$\hbar\dot{\mathbf{r}}_c(t) = \nabla_{\mathbf{k}}\mathcal{E}_{n,\mathbf{k}} - e\mathbf{E}\times\boldsymbol{\Omega}_n(\mathbf{k})$$

Detectable as lateral shift of intensity distribution by Δx perpendicular to F

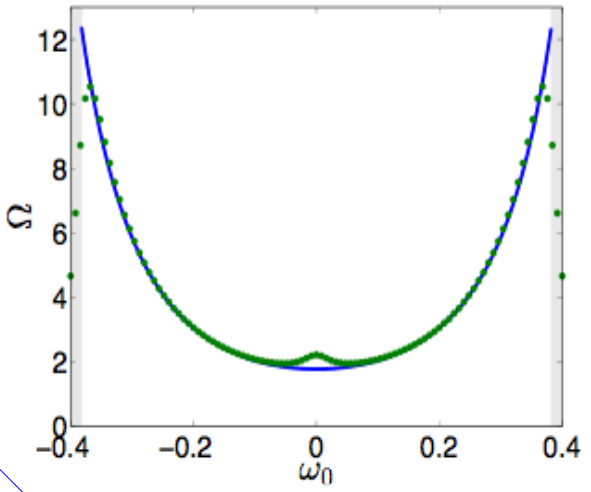
T. Ozawa and IC, *Anomalous and Quantum Hall Effects in Lossy Photonic Lattices*, PRL (2014)
Recently extended to Fubini-Study metric in Ozawa,.arXiv:1708.00333

More quantitatively

		1st	2nd	3rd	4th	5th	6th
$\alpha = \frac{1}{3}$	$\mathcal{C}$	−1	+2	−1			
	$\mathcal{C}_n$	−0.91	-	−0.91			
$\alpha = \frac{1}{5}$	$\mathcal{C}$	−1	−1	+4	−1	−1	
	$\mathcal{C}_n$	−0.97	−0.66*	-	−0.66*	−0.97	
$\alpha = \frac{1}{6}$	$\mathcal{C}$	−1	−1	+2	+2	−1	−1
	$\mathcal{C}_n$	−0.96	−1.06	-	-	−1.06	−0.96
$\alpha = \frac{3}{7}$	$\mathcal{C}$	+2	−5	+2	+2	+2	−5
	$\mathcal{C}_n$	2.05	-	-	2.01	-	-
$\alpha = \frac{4}{9}$	$\mathcal{C}$	+2	+2	−7	+2	+2	+2
	$\mathcal{C}_n$	1.96	-	-	2.02	1.92	2.02
$\alpha = \frac{5}{11}$	$\mathcal{C}$	+2	+2	−9	+2	+2	+2
	$\mathcal{C}_n$	1.92	1.88	-	-	2.06	1.91



(a) Lowest band of $\alpha = 1/3$



(b) Middle band of $\alpha = 1/5$

band gap

Band filling controllable by pumping parameters:

Low loss ($\gamma <$ bandwidth) → $\Delta x = F\,\Omega(k_0)/2\gamma$ (anomalous Hall eff.)

Large loss (bandwidth $< \gamma <$ bandgap) → $\Delta x = q\ \text{Chern} / 2\pi\gamma$ (integer-QH)

Integer quantum Hall effect for photons (in spite of no Fermi level)

Photon phase observable => what about gauge invariance?

T. Ozawa and IC, *Anomalous and Quantum Hall Effects in Lossy Photonic Lattices*, PRL (2014)
Recently extended to Fubini-Study metric in Ozawa,.arXiv:1708.00333

Berry curvature beyond semiclassics

Chang-Niu's semiclassical equations of motion:

$$\hbar\dot{\mathbf{k}}_c(t) = e\mathbf{E}\,,$$
$$\hbar\dot{\mathbf{r}}_c(t) = \nabla_{\mathbf{k}}\mathcal{E}_{n,\mathbf{k}} - e\mathbf{E}\times\mathbf{\Omega}_n(\mathbf{k})$$

Can be derived from minimal-coupling like quantum Hamiltonian: $H = E_n(p) + W[r + A_n(p)]$

where $W(r) = -\,e\,E\,r$ $E_n(p)$ = band dispers. $A_n(p)$=Berry connection

Similar to usual minimal coupling $H = e\,\Phi(r) + [p - e\,A(r)]^2 / 2\,m$ with r ↔ p exchanged

Physical position $r_{ph}=r+A_n(p)$	↔	physical momentum $p - e\,A(r)$
Berry connection $A_n(p)$	↔	magnetic vector potential A(r)
Berry curvature $\Omega_n(p)=\mathrm{curl}_p\, A_n(p)$	↔	magnetic field $B(r)=\mathrm{curl}_r\, A(r)$
band dispersion $E_n(p)$	↔	scalar potential $e\,\Phi(r)$
trap energy W(r)	↔	kinetic energy $p^2/2m$

Price, Ozawa, IC, *Quantum Mechanics Under a Momentum Space Artificial Magnetic Field* , PRL 2014
and references therein (starting from Karplus-Luttinger 1954)

Harper-Hofstadter model + harmonic trap

Magnetic flux per plaquette $\alpha = 1/q$:

- for large q, bands almost flat $E_n(p) \approx E_n$
- lowest bands have $C_n=-1$ and almost uniform Berry curvature $\Omega_n = a^2/2\pi\alpha$

Within single band approximation:

Momentum space magnetic Hamiltonian $H=E_n+ k[r +A_n(p)]^2/2$

equivalent to quantum particle in constant B: $H= c^{te} + [p - e A(r)]^2 / 2 m$

Mass fixed by harmonic trap strength k

- Landau Levels spaced by "cyclotron" → unifom level spacing $k |\Omega_n|$
- Global (toroidal) topology of FBZ matters → LL degeneracy reduced to $|C_n|$

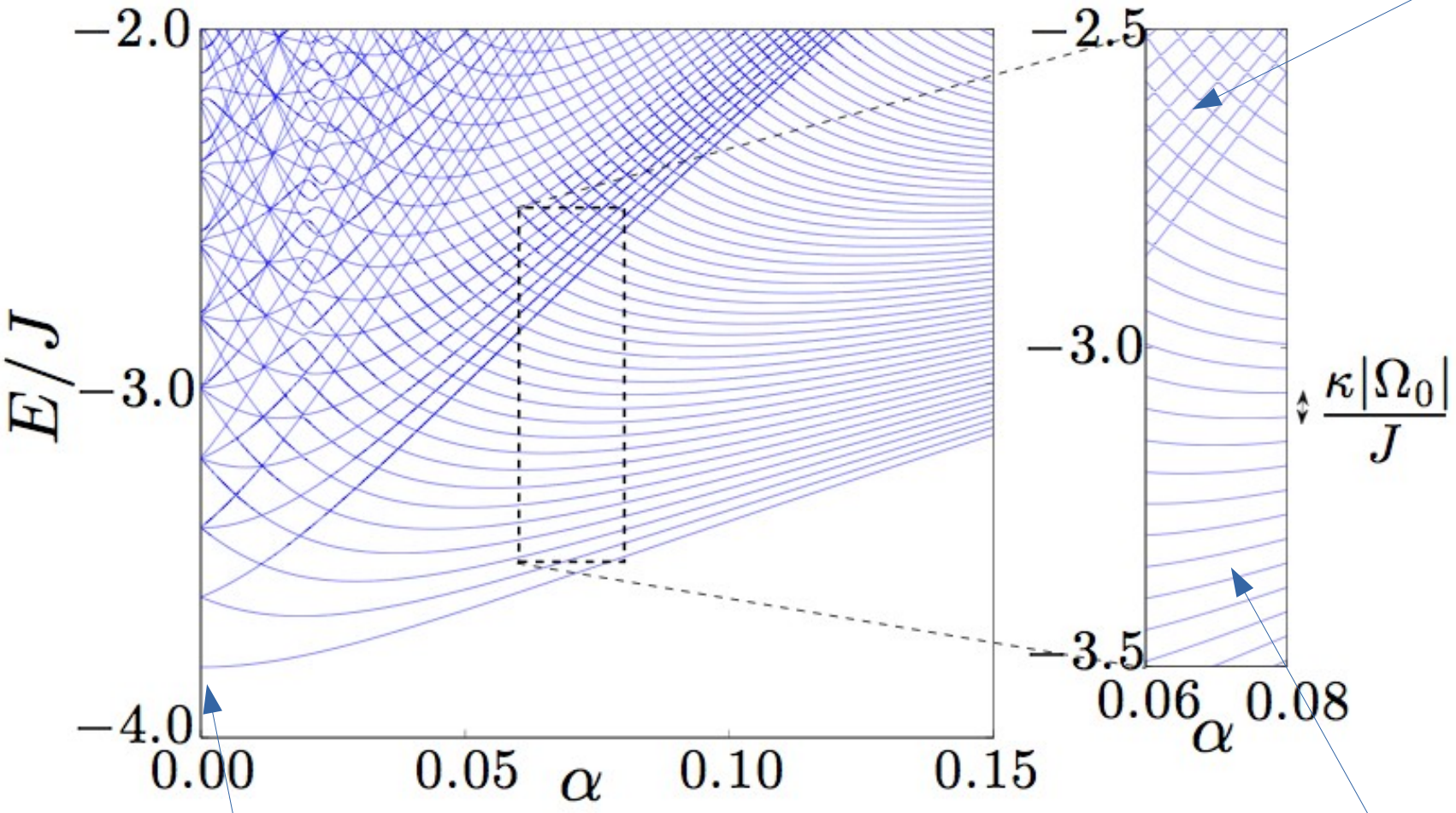
Of course, if:

- Too small α / too strong trap → band too close for single band approx
- Too large α / too weak trap → effect of $E_n(p)$ important

Price, Ozawa, IC, *Quantum Mechanics Under a Momentum Space Artificial Magnetic Field* , PRL 2014

Numerical spectrum

crossing with Landau levels of second HH band



α → 0 harmonic trap states (band gap too small)

Landau levels of lowest HH band

Price, Ozawa, IC, *Quantum Mechanics Under a Momentum Space Artificial Magnetic Field* , PRL 2014

Simulation of realistic photonic experiment

Pumping edge cavity of 11x11 array with realistic parameters from Hafezi et al.

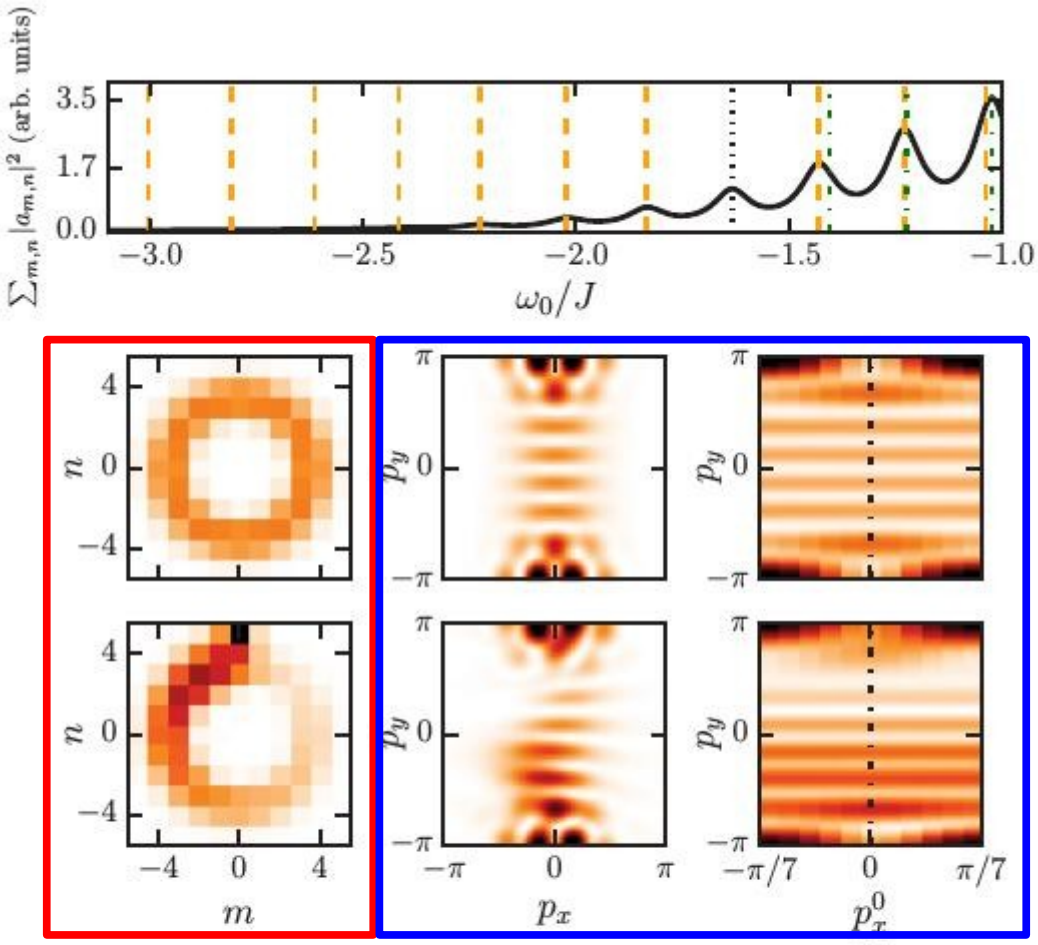
Total transmission spectrum

→ peaks at mode eigenfrequencies

Field profile on resonance

→ eigenstate wavefunction

both in real space (near field)
and in momentum space (far field)



A. C. Berceanu, T. Ozawa, H. M. Price, IC, *Momentum-space Landau levels in driven-dissipative cavity arrays*, PRA 93, 013827 (2016)

2017/18 - Topological lasing

What happens if one adds gain to a topological model ?

First experiments on topological lasing:

- St.Jean et al., Nat. Phot 2017 (1D-SSH model → more in Jacqueline's talks)
- A bit later: Khajavikhan's group, 2017 (1D-SSH); Bahari et al., Science 2017 (2D magnetic photonic crystal)

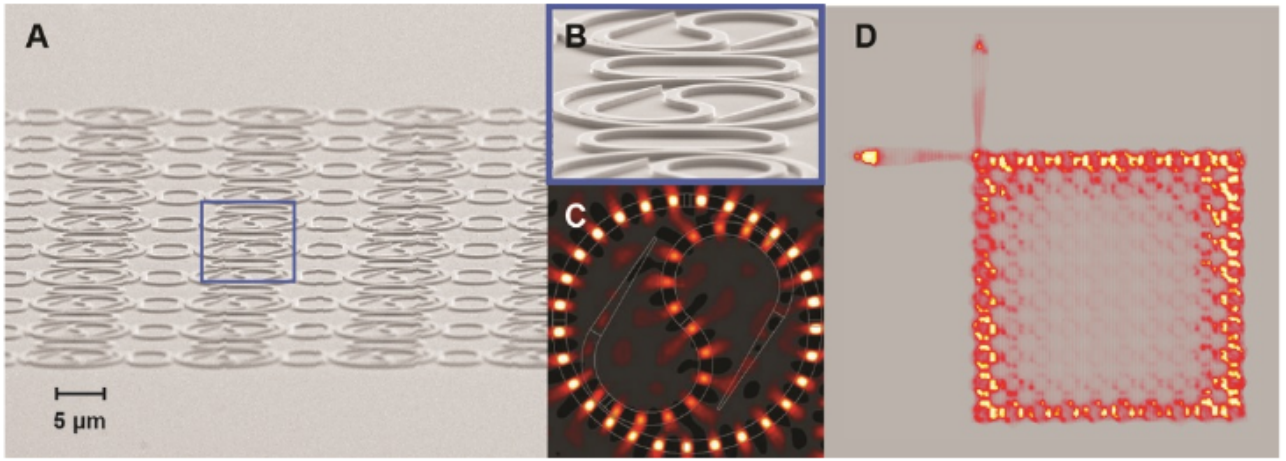


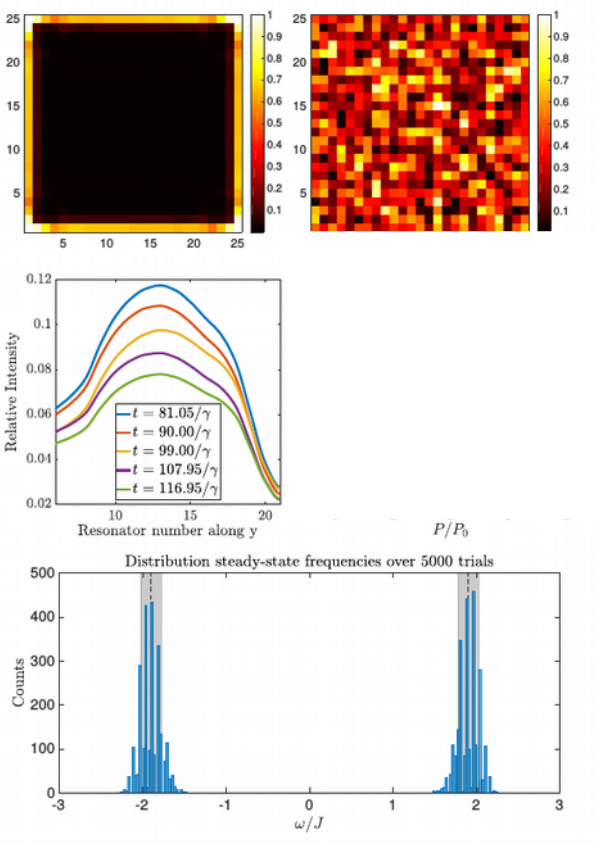
Figure from Bandres et al., Science 2018
Theory in Harari et al., Science 2018

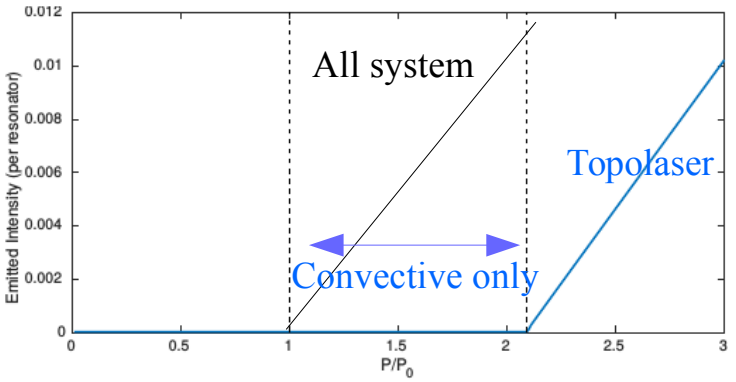
- Hafezi-style array of Si-based ring resonators
- Gain provided by optically pumped III-V layer
- Pump concentrated on edges

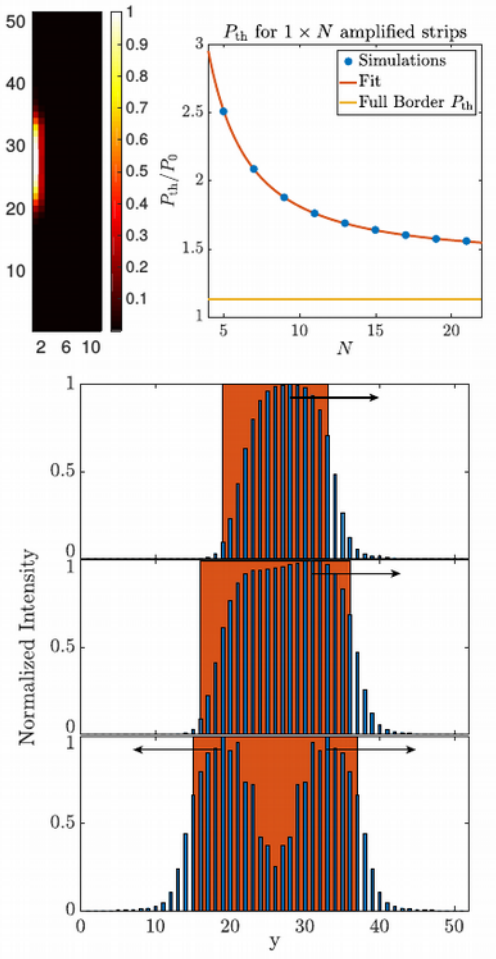
Laser operation into edge mode:

- randomly chooses one direction; immune to disorder
- efficiently funnels pumping energy into single mode laser emission, high slope efficiency (trivial system → pumping many cavities typically gives complicate many-mode emission)

A few intriguing surprises...







Convective vs. absolute instability → different threshold of edge-mode lasing

Topological effects visible in lasing threshold for high number of pumped sites

Topological robustness against mode jumps:

- Random choice of lasing mode over wide bandwidth when all edge pumped
- Extra-slow relaxation of fluctuations.
- Single mode recovered if pumping is spatially interrupted

M. Seclì's MSc thesis, UniTN (Oct. 2017)

Part 2:

Synthetic dimensions

Where we try to make photons explore four spatial dimensions

What about higher dimensions?

Generalize of semiclassical equations to 4D:

$$\begin{cases} \dot{r}^{\mu}(\boldsymbol{k}) = \dfrac{\partial \mathcal{E}(\boldsymbol{k})}{\partial k_{\mu}} - \dot{k}_{\nu}\Omega^{\mu\nu}(\boldsymbol{k}) \\ \dot{k}_{\mu} = -E_{\mu} - \dot{r}^{\nu} B_{\mu\nu}, \end{cases}$$

Integrate current over filled bands:

- 2D quantized Hall current depends on 1st Chern number

$$j^{y} = \frac{E_x}{(2\pi)^2}\int_{T^2} \Omega\, d^2k = \frac{\nu_1}{2\pi} E_x \quad \text{analogous to} \quad j^{y} = \nu \frac{e^2}{h} \quad \text{well known in IQHE}$$

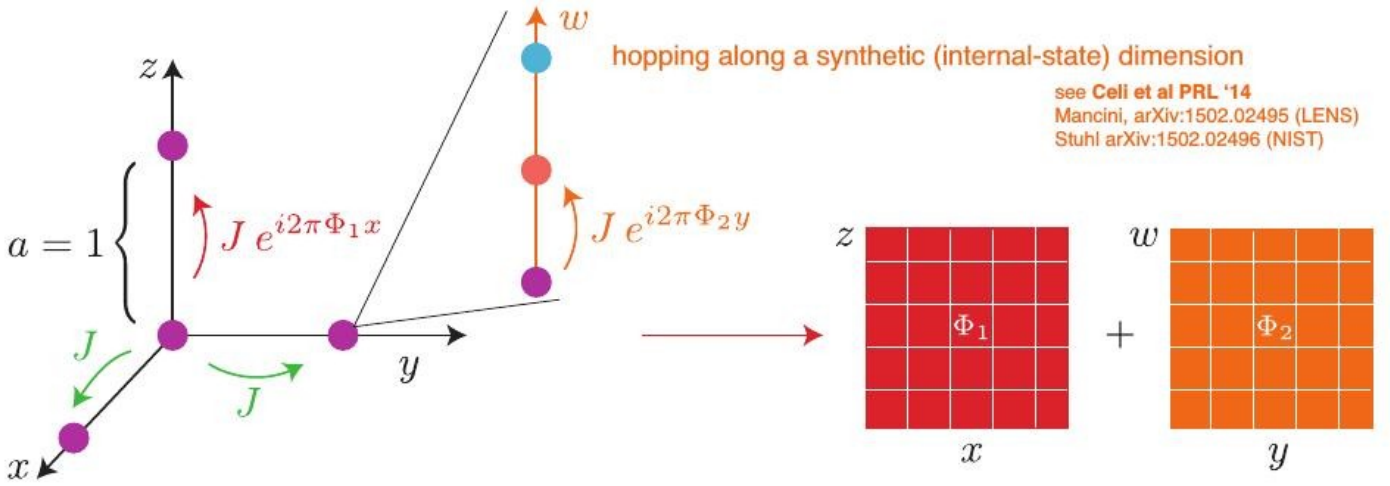
- 4D magneto-electric response depends on 2nd Chern number (non-zero in d≥4)

$$j^{\mu} = E_{\nu}\frac{1}{(2\pi)^4}\int_{\mathbb{T}^4} \Omega^{\mu\nu}\,\mathrm{d}^4k + \frac{\nu_2}{4\pi^2}\varepsilon^{\mu\alpha\beta\nu}E_{\nu}B_{\alpha\beta}$$

$$\nu_2 = \frac{1}{4\pi^2}\int_{\mathbb{T}^4} \Omega^{xy}\Omega^{zw} + \Omega^{wx}\Omega^{yz} + \Omega^{zx}\Omega^{yw}\,\mathrm{d}^4k$$

H. M. Price, O. Zilberberg, T. Ozawa, IC, N. Goldman, *Four-Dimensional Quantum Hall Effect with Ultracold Atoms*, PRL **115**, 195303 (2015);
A bit more abstract: Zhang-Hu, Science 294, 823 (2001); Qi-Hughes-Zhang, Phys. Rev. B 78, 195424 (2008).

How to create 4D system with atoms?



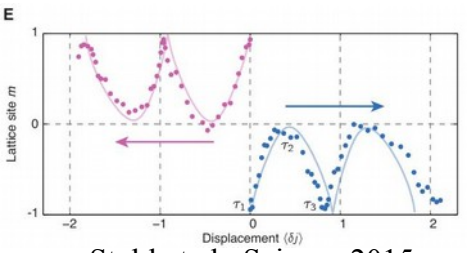
Internal state → Synthetic dimension w

Raman processes give tunneling along *w*

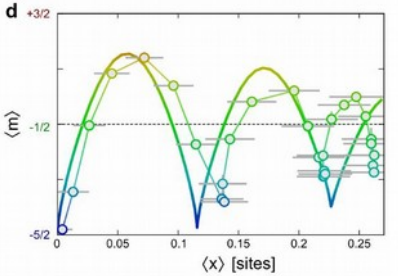
- Spatial phase of Raman beams give Peierls phase in *xw*, *yw*, *zw*
- Standard synthetic-B in *xy* and/or *yz* and/or *zx*

First experimental realizations:

- 1+1 dimens. using 3 spin states
- Cyclotron + Reflection on edges



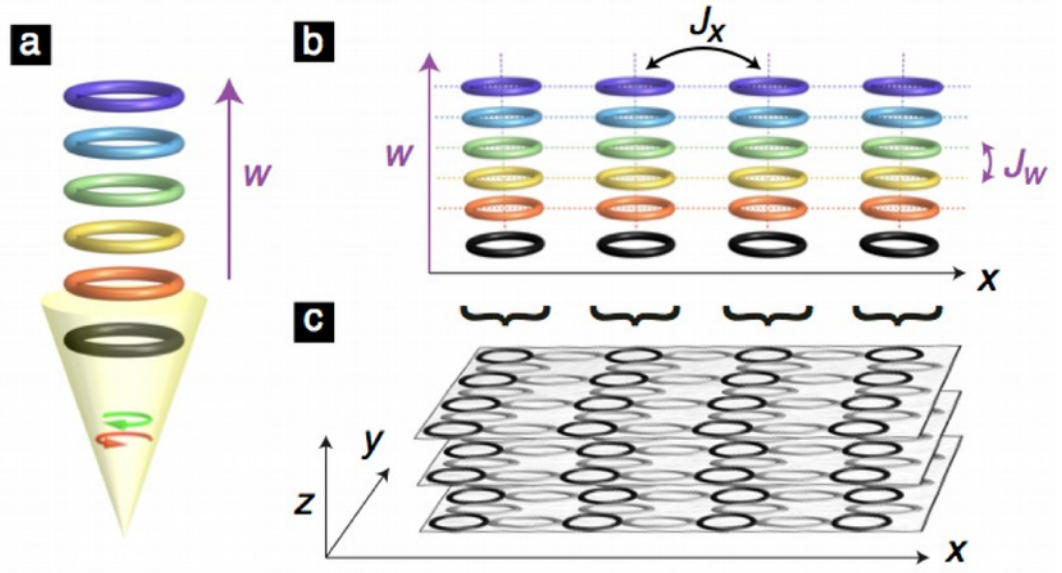
Stuhl et al., Science 2015



Mancini et al., Science 2015.

- Other options → momentum states as synthetic dimension (Gadway)
- Recently: 2D topological pumping in 2D system → analogous to 4D IQHE
  (Lohse, Price, et al., Nature 2018; similar results in photonics in Zilberberg et al., Nature 2018)

How to create synthetic dimensions for photons?



Different modes of ring resonators → synthetic dimension *w*

Tunneling along synthetic *w*:

- strong beam modulates resonator ε_{ij} at ω_{FSR} via optical $\chi^{(3)}$
- neighboring modes get linearly coupled
- phase of modulation → Peierls phase along synthetic *w*

Peierls phase along *x,y,z* → Hafezi's ancilla resonators

Extends Shanhui Fan's idea of synthetic gauge field via time-dependent modulation (Nat. Phys. 2008). See related work by Fan (2015/6)

Differently from atoms: can work with long synthetic dimension *w* with uniform tunneling

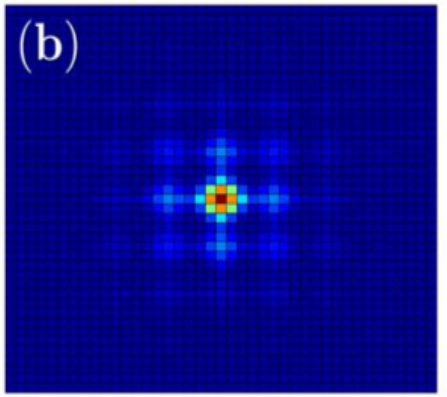
Other schemes possible → orbital angular momentum (Zhou et al.), pulse arrival time (Wimmer-Peschel)

T. Ozawa, N. Goldman, O. Zilberberg, H. M. Price, and IC, *Synthetic Dimensions in Photonic Lattices: From Optical Isolation to 4D Quantum Hall Physics*, PRA 93, 043827 (2016)

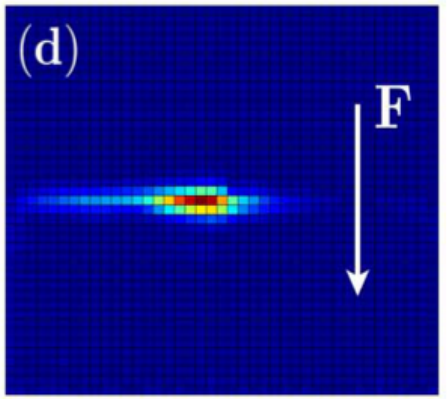
1+1 array: chiral edge states & optical isolation

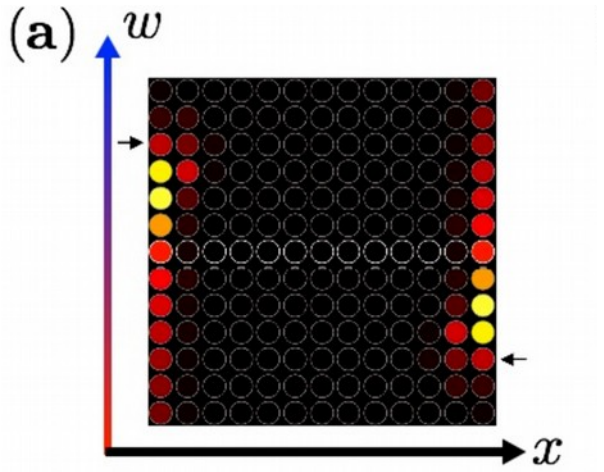
1 (physical) + 1 (synthetic) dimensions: Hofstadter model

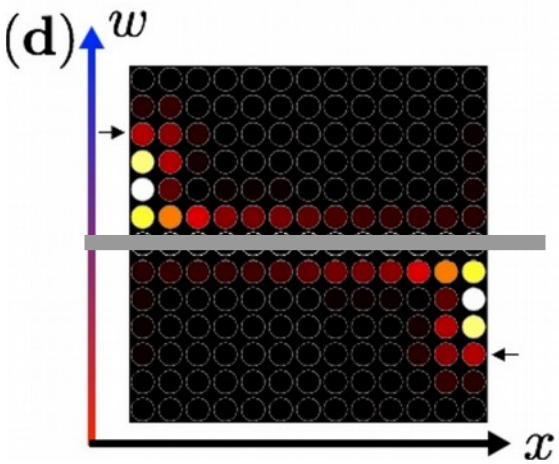
- Bulk topological invariant → Chern number
  - measured via Integer Quantum Hall effect



- Chiral states on edges:
  - Physical edges along x
  - Synthetic edges via design of ε(ω)
    (e.g. inserting absorbing impurities in chosen sites)
    → topologically protected optical isolator







Absorbing row of sites

T. Ozawa, N. Goldman, O. Zilberberg, H. M. Price, and IC, *Synthetic Dimensions in Photonic Lattices: From Optical Isolation to 4D Quantum Hall Physics*, PRA 93, 043827 (2016)

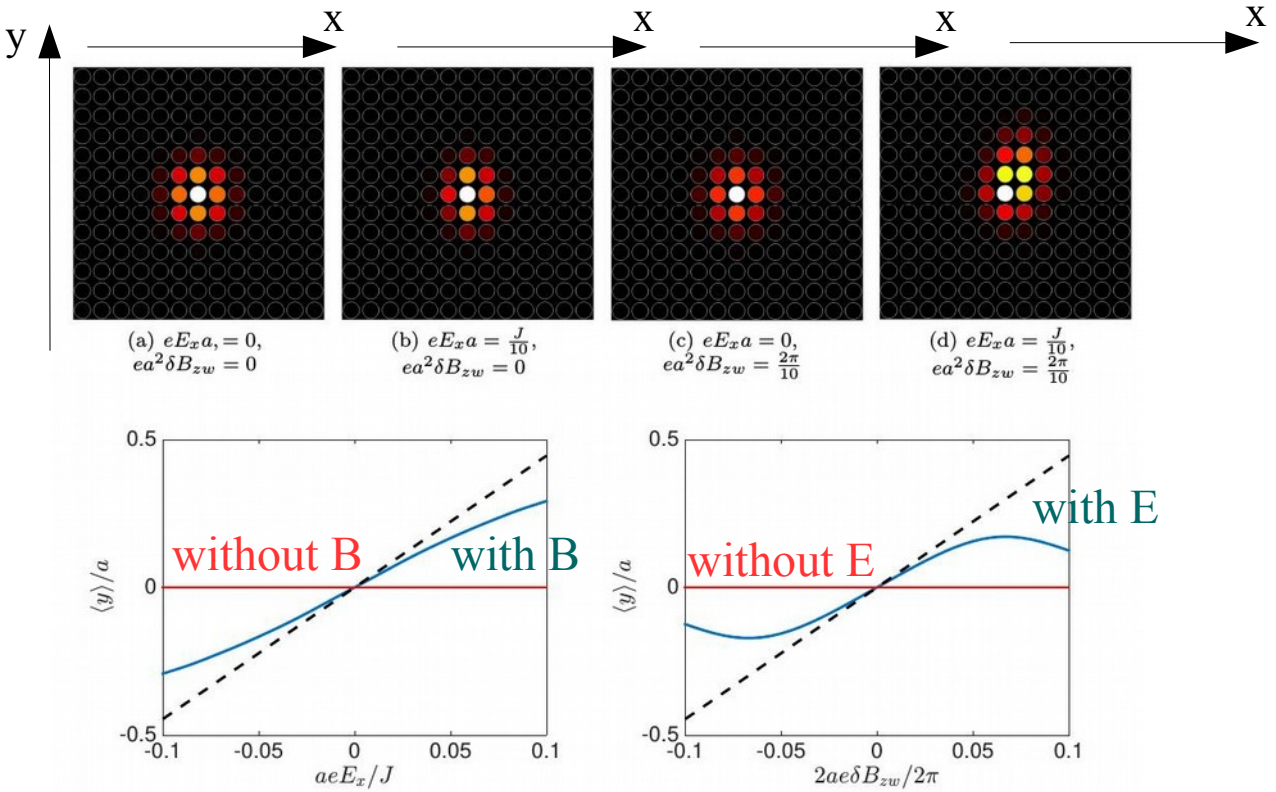
3+1 array: 4D Quantum Hall physics

4D magneto-electric response

Nonlinear integer QH effect

Lateral shift of photon intensity distribution in response to external synth-E and synth-B:

- only present with both E & B
  - proportional to 2nd Chern



$$j^\mu = E_\nu \frac{1}{(2\pi)^4} \int_{\mathbb{T}^4} \Omega^{\mu\nu} \mathrm{d}^4k + \frac{\nu_2}{4\pi^2} \varepsilon^{\mu\alpha\beta\nu} E_\nu B_{\alpha\beta}$$

$$\nu_2 = \frac{1}{4\pi^2} \int_{\mathbb{T}^4} \Omega^{xy}\Omega^{zw} + \Omega^{wx}\Omega^{yz} + \Omega^{zx}\Omega^{yw} \mathrm{d}^4k$$

T. Ozawa, N. Goldman, O. Zilberberg, H. M. Price, and IC, *Synthetic Dimensions in Photonic Lattices: From Optical Isolation to 4D Quantum Hall Physics*, PRA 93, 043827 (2016)

Unfortunately, no time to discuss charge pumping experiments with atoms (Lohse et al. Nature 2018) and light (Zilberberg et al. Nature 2018)

Part 3:

Strongly interacting many-body physics

Towards Fractional Quantum Hall states of light

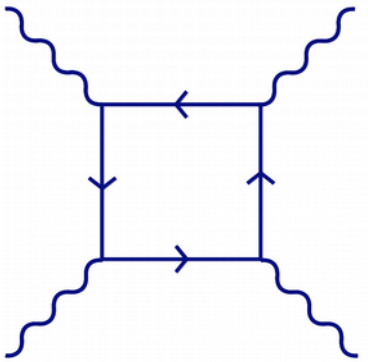
What about interactions?

Photon-photon interactions exist in QED:

Heisenberg-Euler processes via electron-positron exchange

… but cross section ridiculously small for visible light

(recent experiment in accelerator → Nat. Phys. 2017)



$$\sigma \sim \alpha^4 \frac{\hbar^2}{m^2 c^2} \left(\frac{\hbar \omega}{mc^2} \right)^6$$

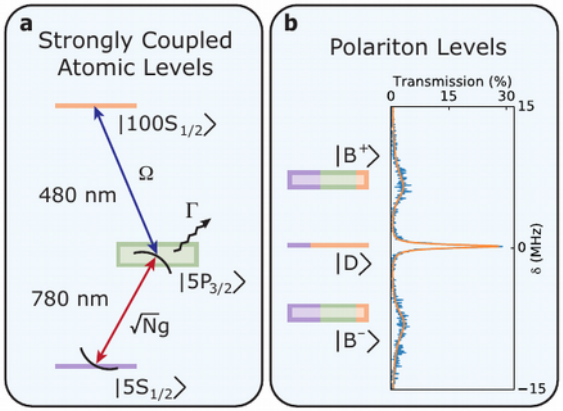
How to enhance it ?

Replace electron-positron pair (E~1MeV) with
electron-hole pair (E~1eV) → gain factor $(10^6)^6 = 10^{36}$!!

In optical language:

- $\chi^{(3)}$ nonlinearity ↔ local photon-photon interactions
- typical material → spatially local (or quasi-local) $\chi^{(3)}$

- notable exception: Rydberg atoms
  → ultra-large and long-range nonlinearity in Rydberg-EIT configuration
  (don't miss Michael's lectures!)

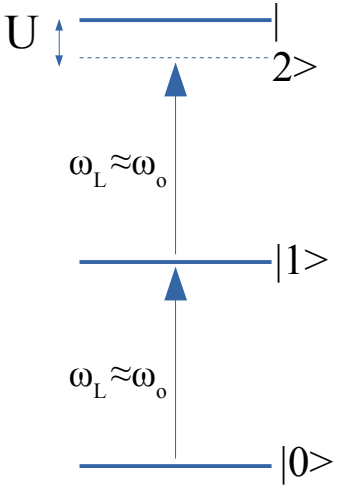


Photon blockade

Bose-Hubbard model:

$$H_0 = \sum_i \hbar\omega_o \hat{b}_i^\dagger \hat{b}_i - \hbar J \sum_{\langle i,j \rangle} \hat{b}_i^\dagger \hat{b}_j + \hbar \frac{U}{2} \sum_i \hat{n}_i(\hat{n}_i - 1)$$

- single-mode cavities at ω_o. Tunneling coupling J
- Polariton interactions: on-site interaction U due to optical nonlinearity



If U >> γ , J, coherent pump resonant with 0→1 transition,
but not with 1→2 transition.

Photon blockade → Effectively impenetrable photons

- Experimentally observed in circuit-QED and Rydberg-EIT,
first evidences in exciton-polaritons (Volz & Imamoglu's groups, 2018)

Need to add driving and dissipation:

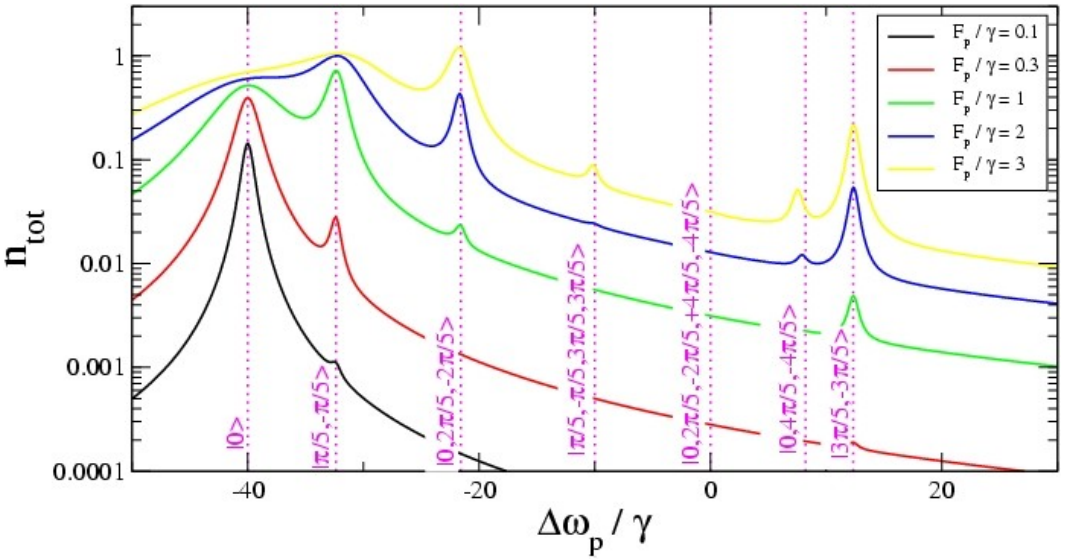
- Incident laser: coherent external driving $\quad H_d = \sum_i F_i(t)\, \hat{b}_i + h.c.$

- Weak losses γ << J, U → Lindblad terms in master eq. determine
non-equilibrium steady-state

Impenetrable "fermionized" photons in 1D necklaces

Many-body eigenstates of Tonks-Girardeau gas of impenetrable photons

Coherent pump selectively addresses specific many-body states



Transmission spectrum as a function pump frequency for fixed pump intensity:

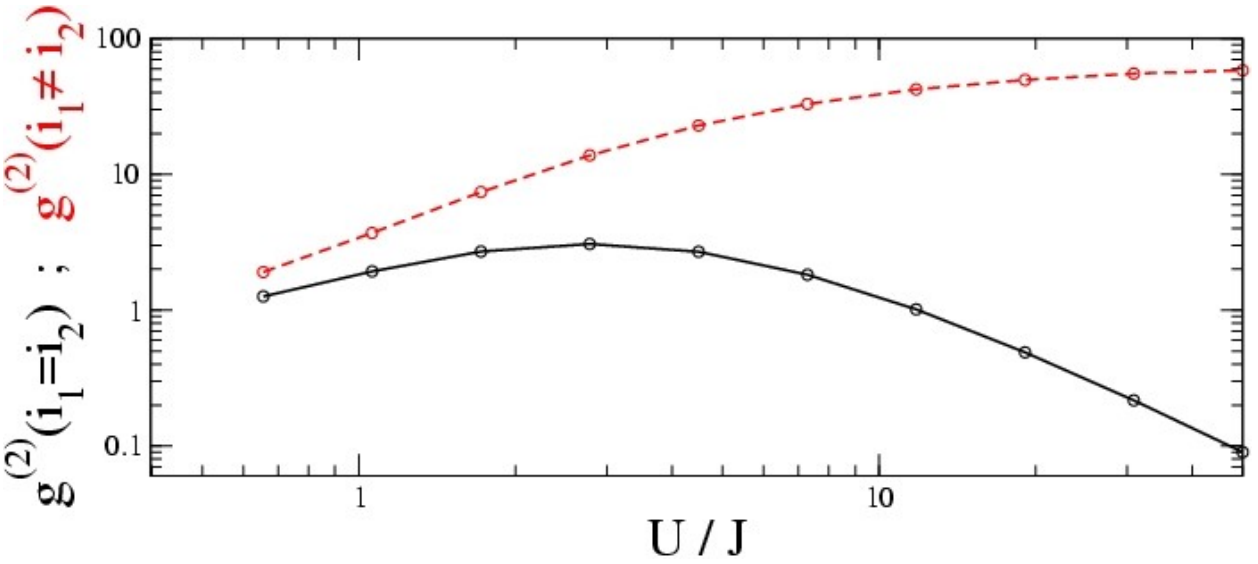
- each peak corresponds to a Tonks-Girardeau many-body state $|q_1,q_2,q_3...>$
- q_i quantized according to PBC/anti-PBC depending on N=odd/even
- U/J >> 1: efficient photon blockade, impenetrable photons.

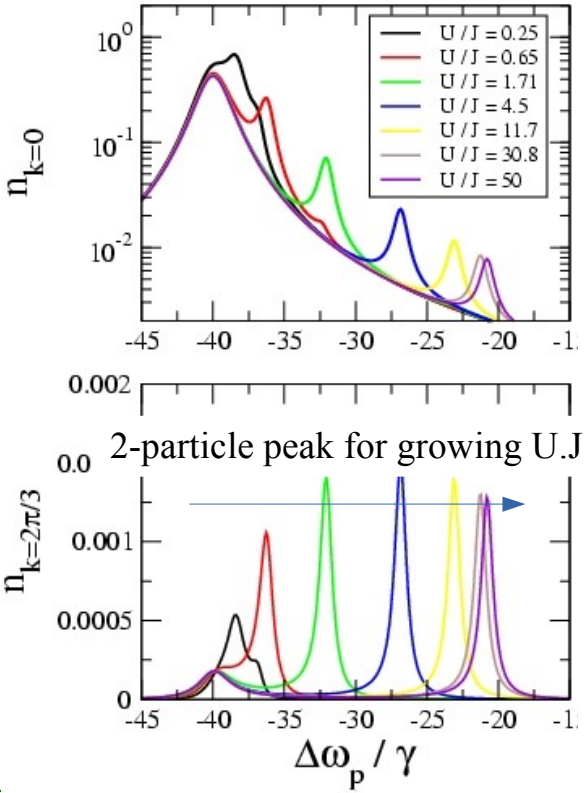
N-particle state excited by N photon transition:

- Plane wave pump with k_p=0: selects states of total momentum P=0
- Monochromatic pump at ω_p: resonantly excites states of many-body energy E such that ω_p= E / N

IC, D. Gerace, H. E. Türeci, S. De Liberato, C. Ciuti, A. Imamoglu, PRL **103**, 033601 (2009)
See also related work D. E. Chang et al, Nature Physics (2008)

State tomography from emission statistics





Finite U/J, pump laser tuned on two-photon resonance

- intensity correlation between the emission from cavities i_1, i_2
- at large U/γ, larger probability of having N=0 or 2 photons than N=1
  - low U<<J: bunched emission for all pairs of i_1, i_2
  - large U>>J: antibunched emission from a single site
    positive correlations between different sites
- Idea straightforwardly extends to more complex many-body states.

Photon blockade + synthetic gauge field = QHE for light

Bose-Hubbard model: $$H_0 = \sum_i \hbar\omega_\circ \hat{b}_i^\dagger \hat{b}_i - \hbar J \sum_{\langle i,j\rangle} \hat{b}_i^\dagger \hat{b}_j e^{i\varphi_{ij}} + \hbar \frac{U}{2} \sum_i \hat{n}_i(\hat{n}_i - 1)$$

gauge field gives phase in hopping terms

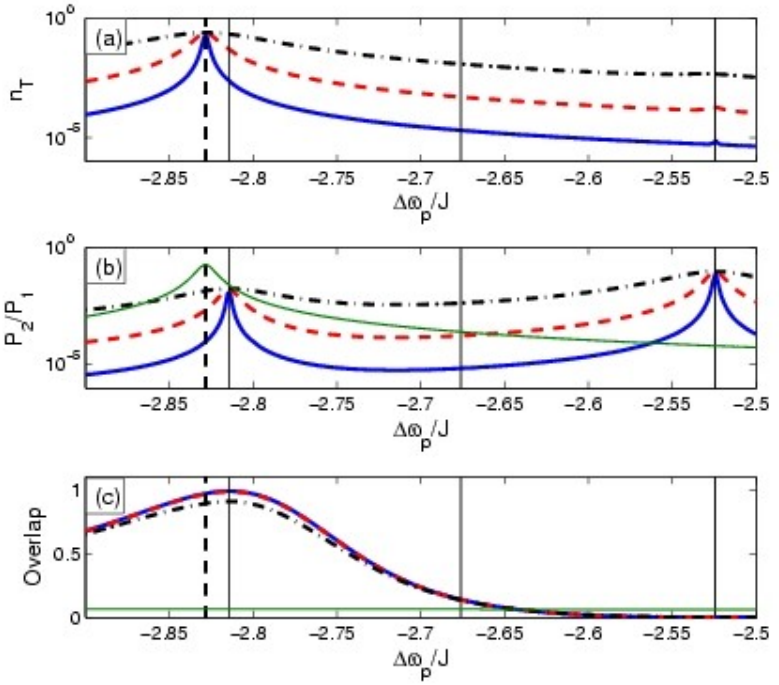
with usual coherent drive and dissipation → look for non-equil. steady state

Transmission spectra:

- peaks correspond to many-body states
- comparison with eigenstates of H_0
- good overlap with Laughlin wf (with PBC)

$$\psi_l(z_1, ..., z_N) = \mathcal{N}_L F_{\rm CM}^{(l)}(Z) e^{-\pi\alpha\sum_i y_i^2} \times \prod_{i<j}^{N} \left(\vartheta\left[\begin{matrix}\frac{1}{2}\\ \frac{1}{2}\end{matrix}\right]\left(\frac{z_i - z_j}{L}\middle| i\right)\right)^2$$

- no need for adiabatic following, etc....

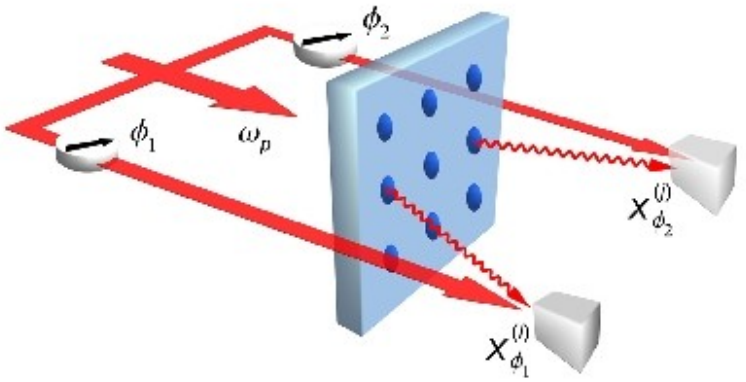


R. O. Umucalilar and IC, *Fractional quantum Hall states of photons in an array of dissipative coupled cavities*, PRL 108, 206809 (2012)
See also related work by Cho, Angelakis, Bose, PRL 2008; Hafezi et al. NJP 2013; arXiv:1308.0225

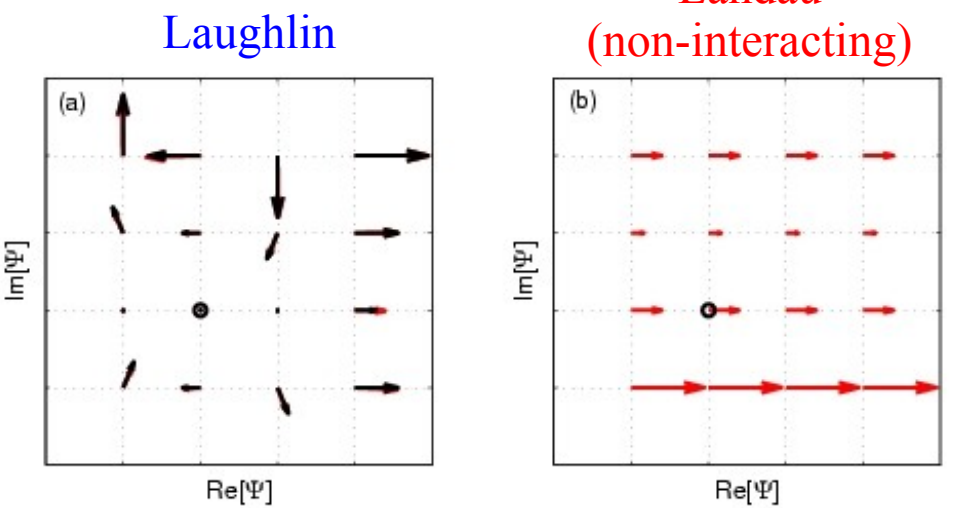
Tomography of FQH states

Homodyne detection of secondary emission

$$\langle \hat{b}_i \hat{b}_j \rangle = \langle X_0^{(i)} X_0^{(j)} \rangle - \langle X_{\pi/2}^{(i)} X_{\pi/2}^{(j)} \rangle + i\langle X_0^{(i)} X_{\pi/2}^{(j)} \rangle + i\langle X_{\pi/2}^{(i)} X_0^{(j)} \rangle$$



Non-trivial structure of Laughlin state compared to non-interacting photons



R. O. Umucalilar and IC, *Fractional quantum Hall states of photons in an array of dissipative coupled cavities*, PRL 108, 206809 (2012)

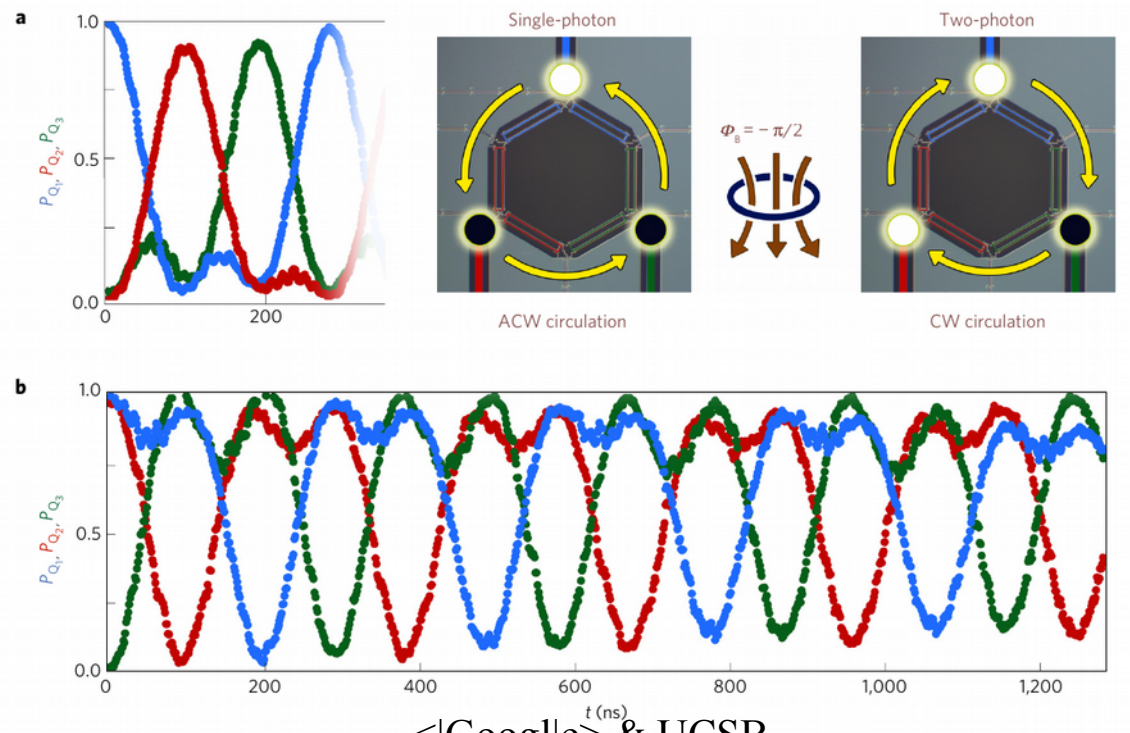
Circuit-QED experiment

Ring-shaped array of qubits

- Transmon qubit: two-level system
  → Impeetrable photons
- Time-modulation of couplings
  → synthetic gauge field

Initialize independently sites

Unitary evolution (until bosons lost)



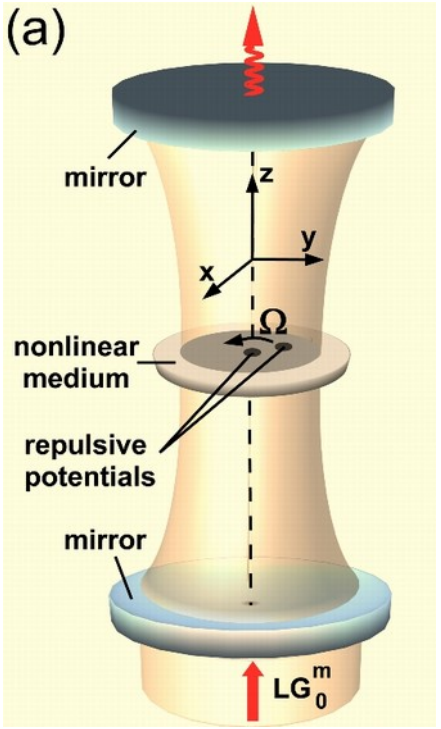
<|Googl|e> & UCSB
Roushan et al., Nat. Phys. 2016
More in Pedram's talk!

"Many"-body effect:

two-photon state → opposite rotation compared to one-photon state
(similar to cold-atom experiment in Greiner's lab, see Tai et al. Nature 2017)

Continuous space FQH physics

Single cylindrical cavity. No need for cavity array



same form → Coriolis $F_c = -2m\Omega \times v$

same form → Lorentz $F_L = e\, v \times B$

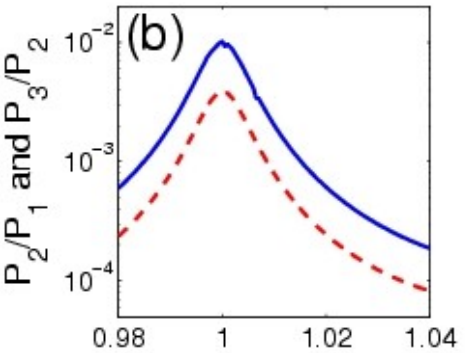
Photon gas injected by Laguerre-Gauss pump

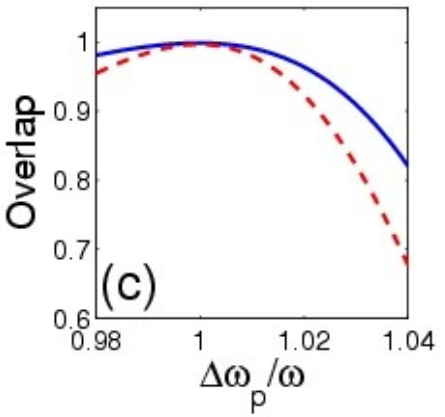
with finite orbital angular momentum

Strong repulsive interactions from nonlinearity

Resonant peak in transmission due to Laughlin state:

$$\psi(z_1,\ldots,z_N) = e^{-\sum_i |z_i|^2/2} \prod_{i<j} (z_i - z_j)^2$$





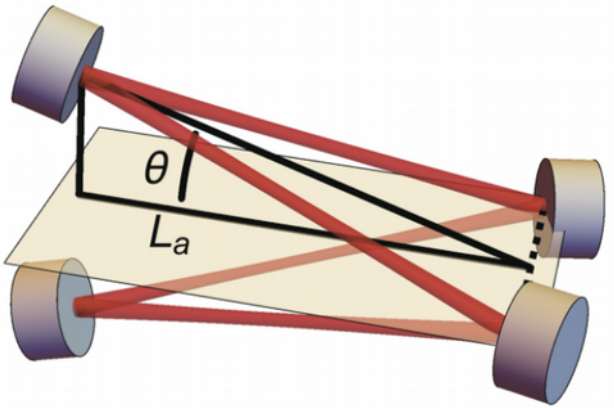
Overlap measured from quadrature noise of transmitted light

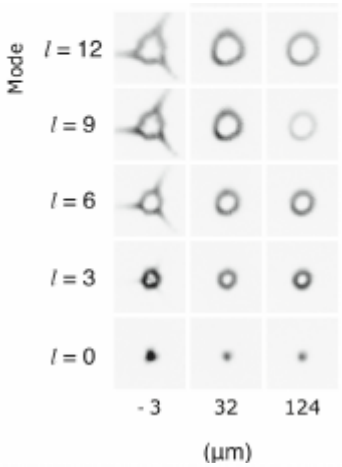
$$\langle \hat{b}_i \hat{b}_j \rangle = \langle X_0^{(i)} X_0^{(j)} \rangle - \langle X_{\pi/2}^{(i)} X_{\pi/2}^{(j)} \rangle + i\langle X_0^{(i)} X_{\pi/2}^{(j)} \rangle + i\langle X_{\pi/2}^{(i)} X_0^{(j)} \rangle$$

R. O. Umucalilar and IC, *Anyonic braiding phases in a rotating strongly correlated photon gas*, arXiv:1210.3070

Experiment @ Chicago

A far smarter design



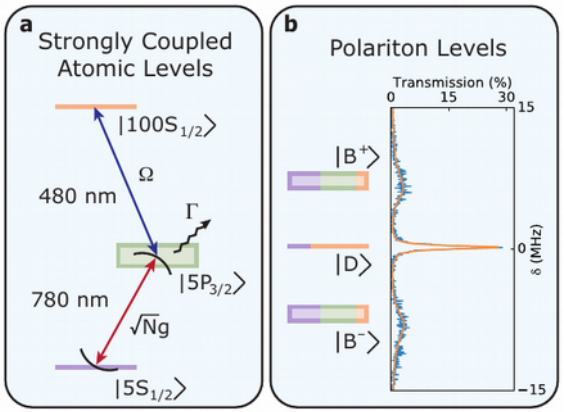


Non-planar ring cavity:

- Parallel transport→ synthetic B
- Landau levels for photons observed

Crucial advantages:

- Narrow frequency range relevant
- Integrated with Rydberg-EIT reinforced nonlinearities



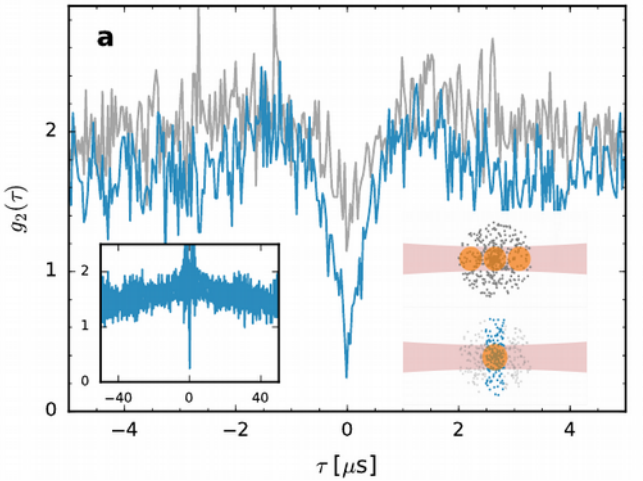
Polariton blockade on lowest (0,0) mode

- Equivalent to $\Delta_{\text{Laughlin}} > \gamma$: Laughlin physics coming soon!

Easiest strategy for Laughlin

- Coherent pumping→ multi-photon peaks to few-body states
- Laughlin state → quantum correlations between orbital modes

(Umucalilar-Wouters-IC, PRA 2014)

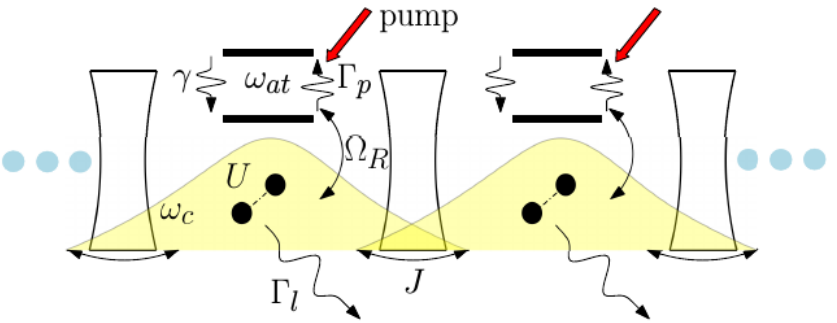


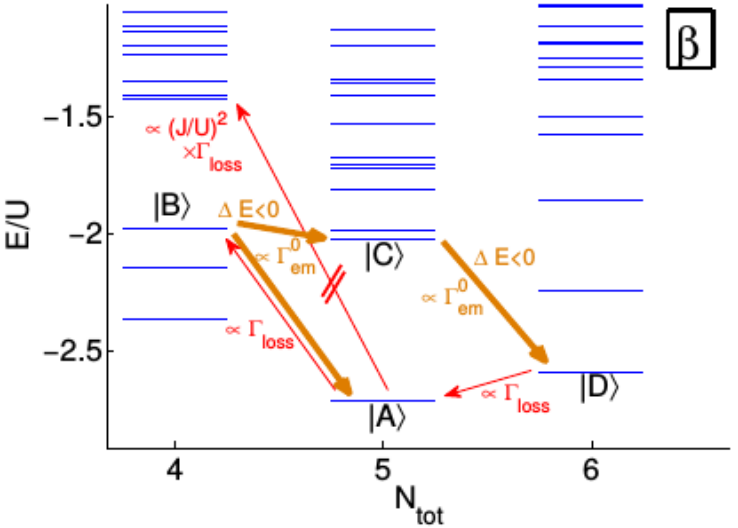
Figures from J. Simon's group @ U. Chicago
Schine et al., Nature 2016; Jia et al. 1705.07475

How to access larger particle numbers

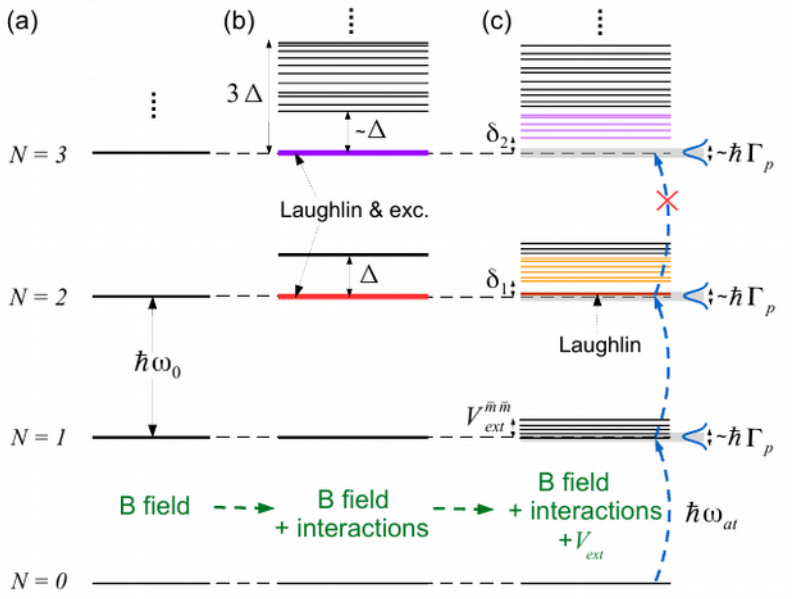
Frequency-dependent incoherent pumping, e.g. collection of inverted emitters

- Lorentzian emission line around ω_{at}
  sophisticated schemes → other spectral shapes
- Emission only active if many-body transition
  is near resonance
- Injects photons until band is full (MI) or
  many-body gap develops (FQH)
- Many-body gap blocks excitation of higher bands





Lebreuilly, Biella, Storme, Rossini, et al., PRA 2017
Biella, Storme, Lebreuilly, Rossini et al., PRA 2017

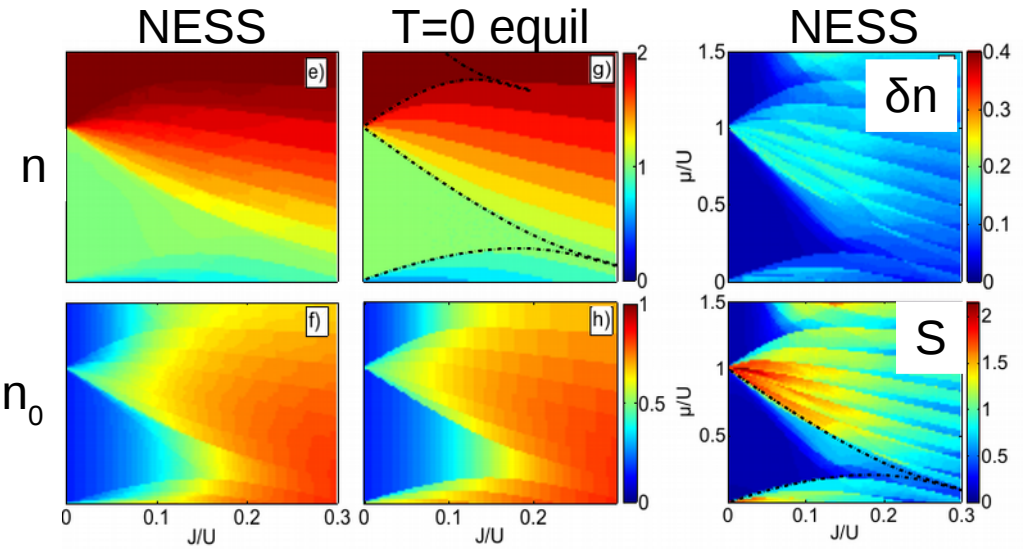


Umucalilar-IC, PRA 2017

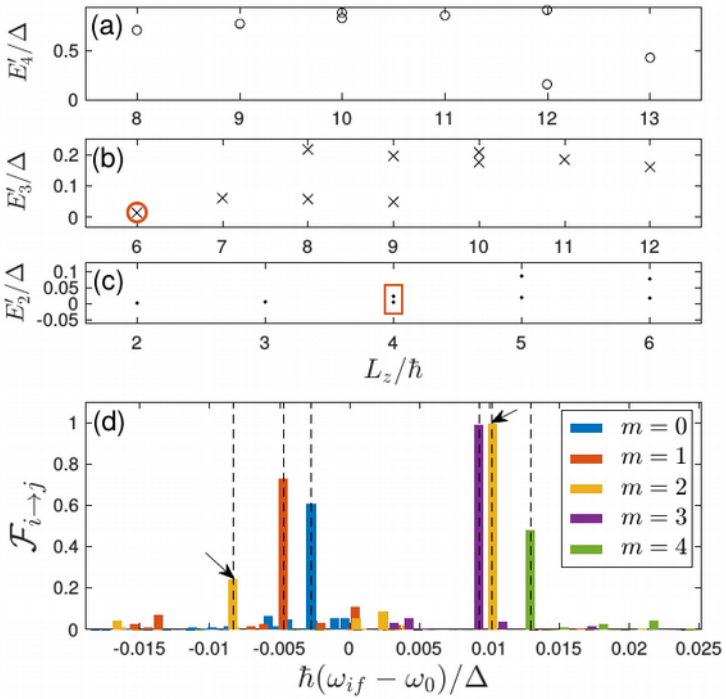
Lebreuilly et al. CRAS (2016)
Kapit, Hafezi, Simon, PRX 2014

Numerical validation for MI & FQH

- Driven-dissipative steady state under non-Markovian master equation
- resembles low-T equilibrium (but interesting deviations in some cases)
- stabilizes strongly correlated many-body states, e.g. Mott-insulator, FQH…
- no restriction to small photon numbers
- radiative emission (lines & spatial correlations) give info on many-body states
- works not only for periodic boundary conditions; FQH gapless edges more fragile, but also stabilized.
  Crucial to study edge physics beyond Luttinger liquid (Macaluso-IC, PRA 2017 + PRA 2018)



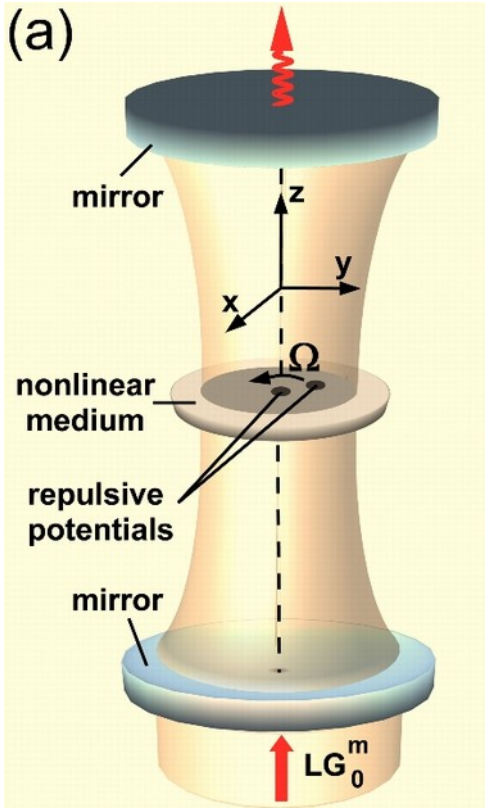
Lebreuilly, Biella, Storme, Rossini et al., 1704.01106 & 1704.08978



Umucalilar-IC, PRA 2017

See also Kapit, Hafezi, Simon, PRX 2014 → PBC case
Other "flux insertion" schemes to create FQH of photons
→ Fleischhauer, Simon et al. & Dutta-Mueller, 2018

Theorists' speculations: anyonic braiding phase



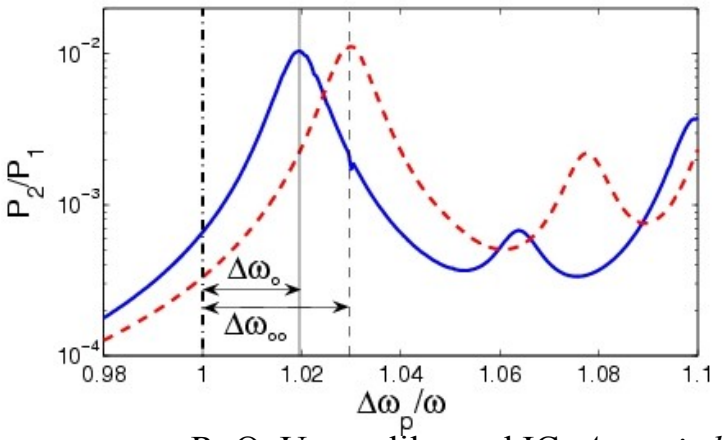
Holes in quantum Hall liquid:

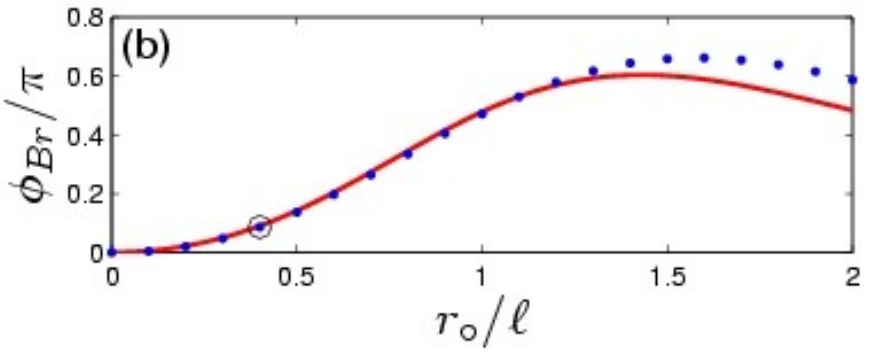
- predicted intermediate statistics between bosons and fermions
- understandable as magnetic flux attached to hole.

So far, no unambiguous experimental proof with electrons.

- LG pump to create and maintain quantum Hall liquid
- Localized repulsive potentials in trap:
  → create quasi-hole excitation in quantum Hall liquid
  → position of holes adiabatically braided in space
- Anyonic statistics of quasi-hole: many-body Berry phase ϕ_{Br} when positions swapped during braiding
- Many-body Berry phase → shift of transmission resonance while repulsive potential moved with period T_{rot} along circle

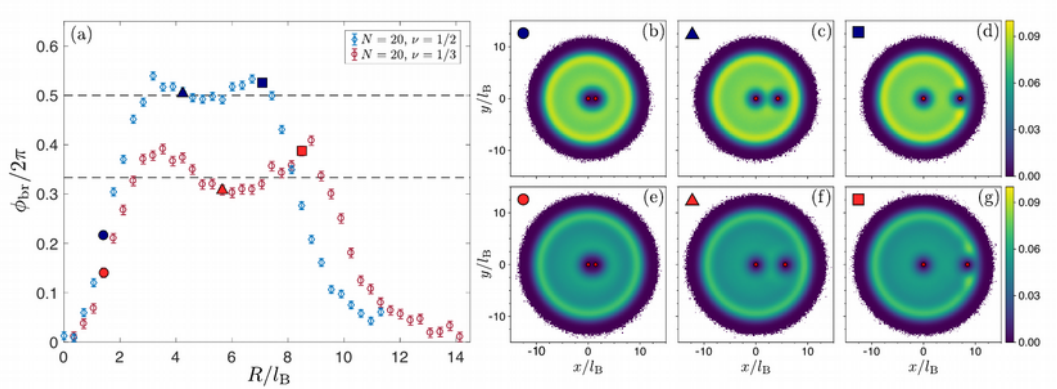
$$\phi_{Br} \equiv (\Delta\omega_{oo} - \Delta\omega_{o})\, T_{rot} \quad [2\pi]$$





R. O. Umucalilar and IC, *Anyonic braiding phases in a rotating strongly correlated photon gas*, arXiv:1210.3070

Observing anyonic statistics via time-of-flight measurements



Braiding phase → Berry phase when two quasi-holes are moved around each other

$$\varphi_{\rm B}(R) = i \oint_R \langle \Psi(\theta)|\partial_\theta|\Psi(\theta)\rangle d\theta$$

Braiding operation can be generated by rotations, so braiding phase related to L_z

$$\varphi_{\rm B}(R) = \frac{1}{\hbar}\oint_R \langle \Psi(\theta)|L_z|\Psi(\theta)\rangle d\theta = \frac{2\pi}{\hbar}\langle L_z\rangle$$

Self-similar expansion of lowest-Landau-levels → L_z can be measured in time-of-flight via size of the expanding cloud

$$\langle r^2\rangle_{\rm tof} = \frac{1}{N}\left(\frac{\hbar t}{\sqrt{2}Ml_B}\right)^2\left(\frac{\langle L_z\rangle}{\hbar}+N\right) = \left(\frac{\hbar t}{2Ml_B^2}\right)^2\langle r^2\rangle$$

Can be applied to both cold atoms or to fluids of light looking at far-field emission pattern

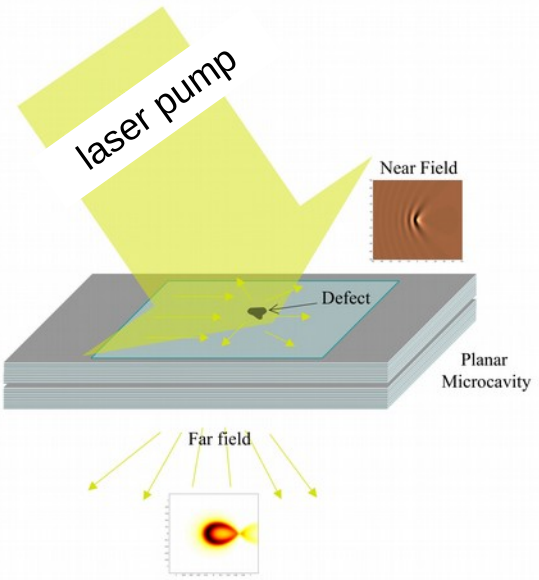
Umucalilar, Macaluso et al., PRL 2018.

Part 4:

Quantum fluids of light with a *unitary* dynamics

Field equation of motion

Planar microcavities & cavity arrays



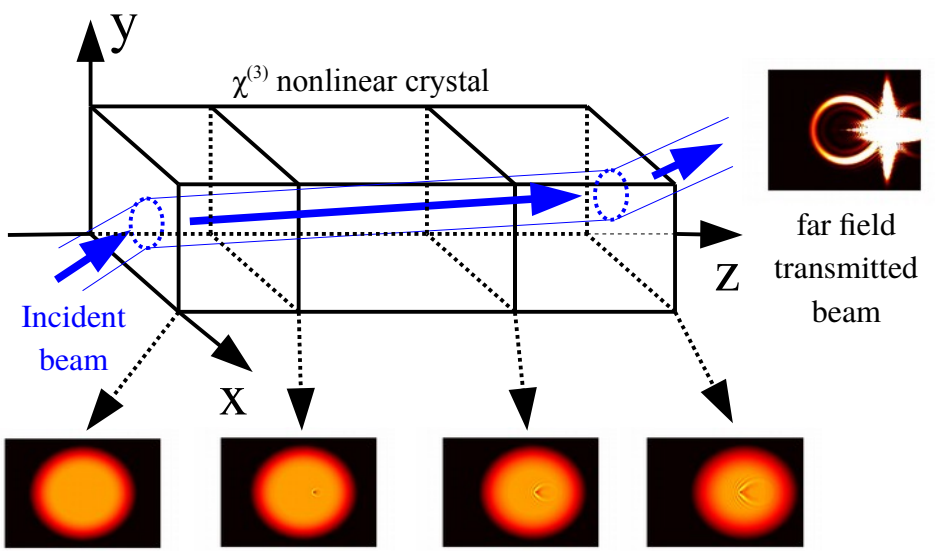
Pump needed to compensate losses:
driven-dissipative dynamics in real time
stationary state ≠ thermodyn. equilibrium

Driven-dissipative CGLE evolution

$$i\frac{dE}{dt}=\left\{\omega_o-\frac{\hbar\nabla^2}{2m}+V_{ext}+g|E|^2+\frac{i}{2}\left(\frac{P_0}{1+\alpha|E|^2}-\gamma\right)\right\}E+F_{ext}$$

Quantum correl. sensitive to dissipation

Propagating geometry



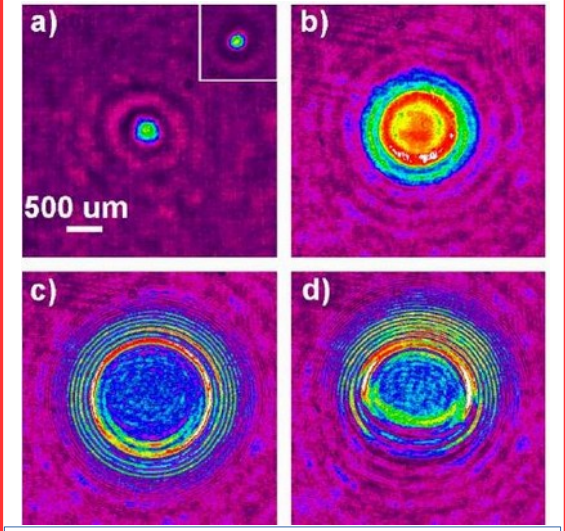
Monochromatic beam
Incident beam sets initial condition @ z=0
MF → Conserv. paraxial propag. → GPE

$$i\frac{dE}{dz}=\left\{-\frac{\hbar\nabla_{xy}^2}{2\beta}+V_{ext}+g|E|^2\right\}E$$

- V_{ext} , g proportional to -(ε(r)-1) and $\chi^{(3)}$
- Mass → diffraction (xy)

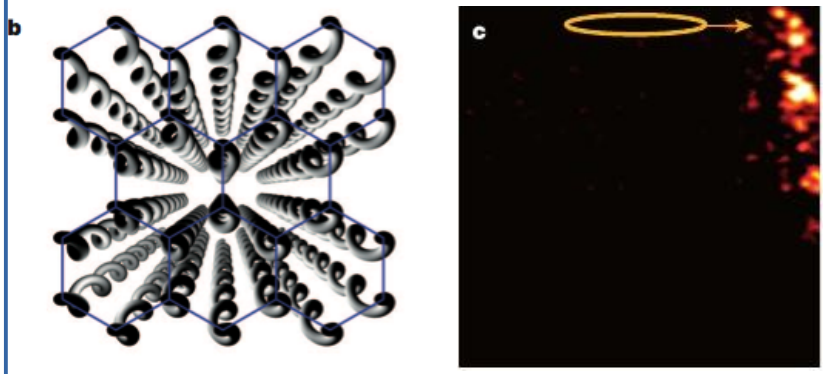
First expts with (almost) conservative QFL's

Dispersive superfluid-like shock waves



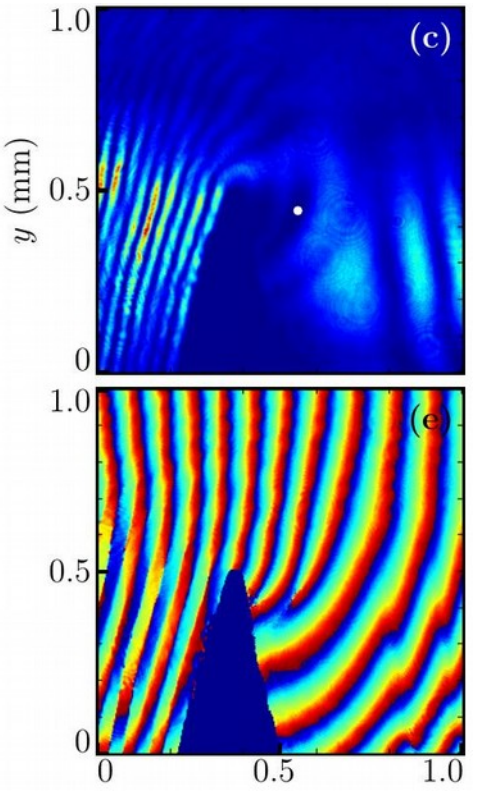
Wan et al., Nat. Phys. 3, 46 (2007)

Chiral edge states in (photonic) Floquet topological insulator

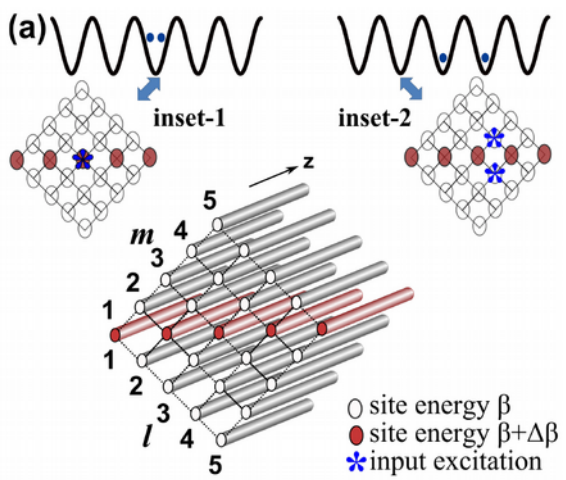


Rechtsman, et al., Nature 496, 196 (2013)

Hydrodynamic nucleation of quantized vortices

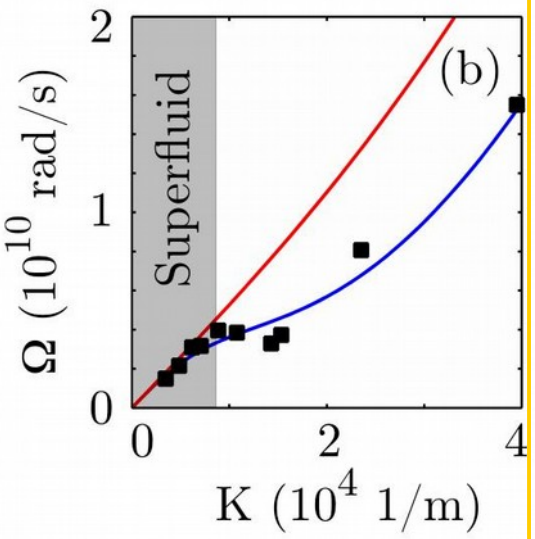


D. Vocke et al., arXiv:1511.06634



Quantum simul. of 2-body physics
Mukherjee et al., arXiv:1604.00689

Bogoliubov dispersion of collective excitations



D. Vocke et al. Optica (2015)

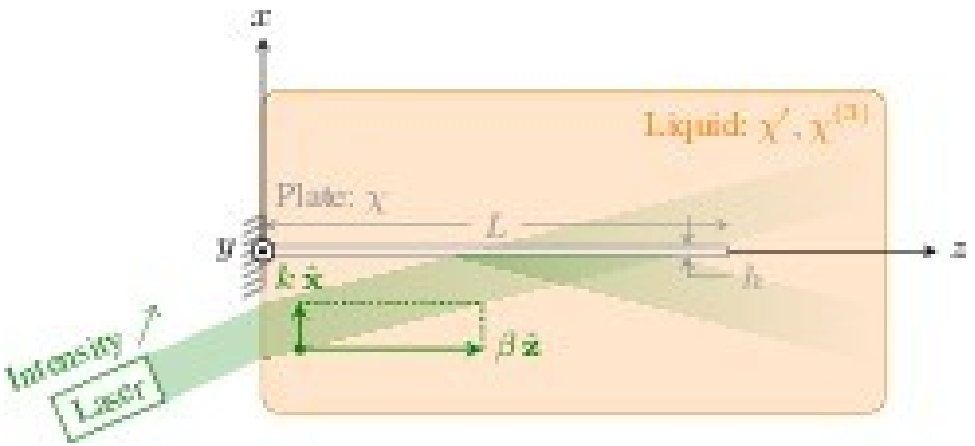
Frictionless flow of superfluid light (I)

All superfluid light experiments so far:

- Planar microcavity device with stationary obstacle in flowing light
- Measure response on the fluid density/momentum pattern
- Obstacle typically is defect embedded in semiconductor material
- Impossible to measure mechanical friction force exerted onto obstacle

Propagating geometry more flexible:

- Obstacle can be solid dielectric slab with different refractive index
- Immersed in liquid nonlinear medium, so can move and deform
- Mechanical force measurable from magnitude of slab deformation



P.-E. Larré, IC, *Optomechanical Signature of a Frictionless Flow of Superfluid Light*, Phys. Rev. A 91, 053809 (2015).

Frictionless flow of superfluid light (II)

Numerics for propagation GPE of monochromatic laser:

$$i\partial_z E = -\frac{1}{2\beta}\left(\partial_{xx}+\partial_{yy}\right)E + V(r)E + g\,|E|^2 E$$

with $V(r) = -\beta\,\Delta\varepsilon(r)/(2\varepsilon)$ with rectangular cross section and $g = -\beta\,\chi^{(3)}/(2\varepsilon)$

For growing light power, superfluidity visible:

- Intensity modulation disappears
- Suppression of opto-mechanical force

An intermediate powers:

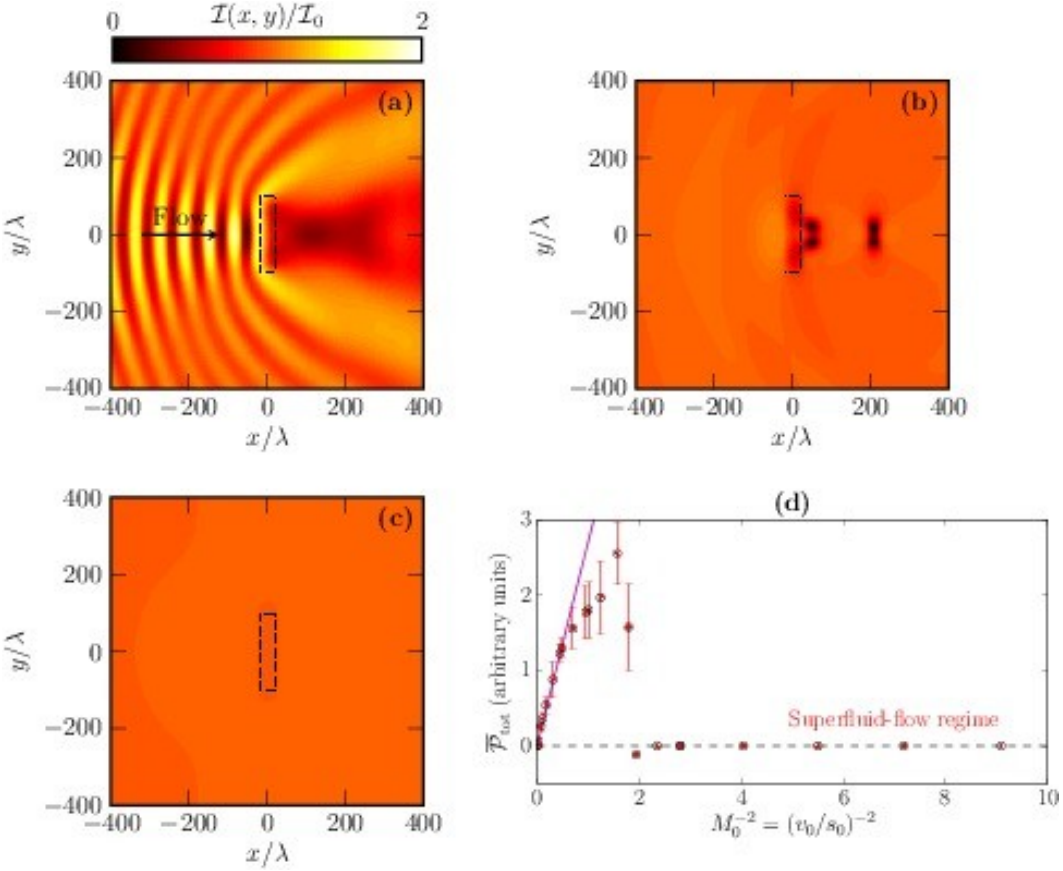
- Periodic nucleation of vortices
- Turbulent behaviours

Fused silica slab as obstacle

→ deformation almost in the μm range

Experiment in progress

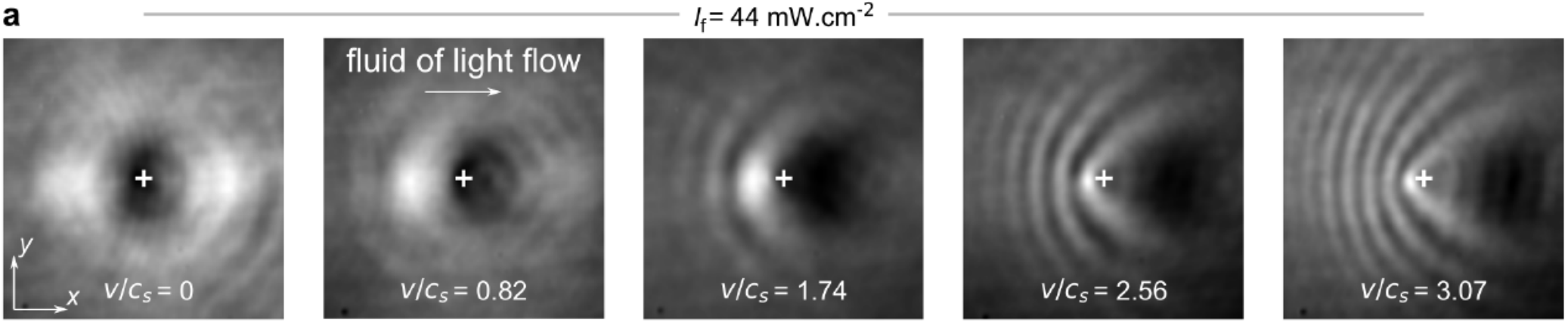
→ surrounding medium in fluid state
but local nonlinearity (e.g. atomic gas)



P.-E. Larré, IC, *Optomechanical Signature of a Frictionless Flow of Superfluid Light*, Phys. Rev. A 91, 053809 (2015).

U-Nice experiments: Michel et al., Nat. Comm. 2018

Increasing incidence angle, i.e. speed

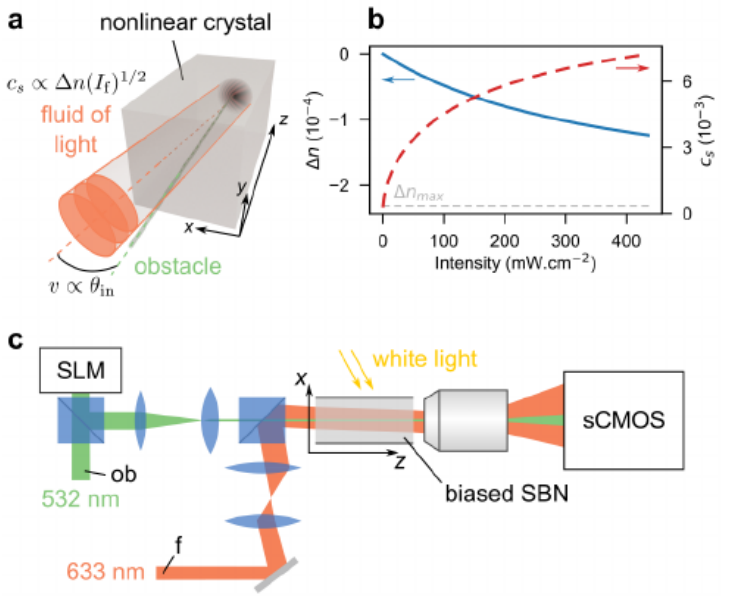


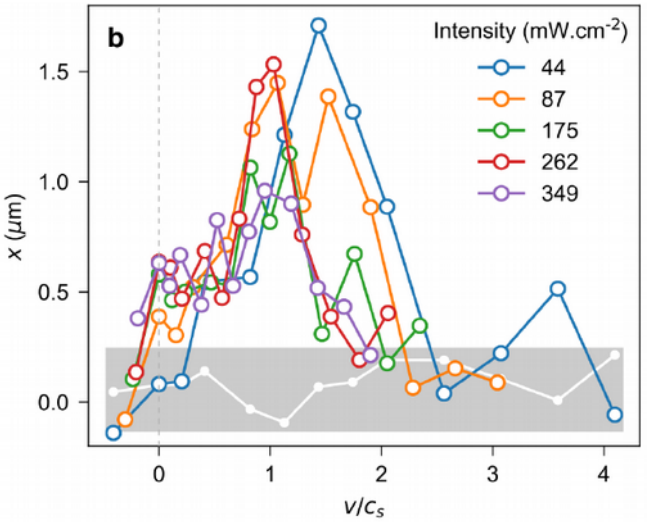
Superfluid

Cherenkov cone shrinks with v/c_s

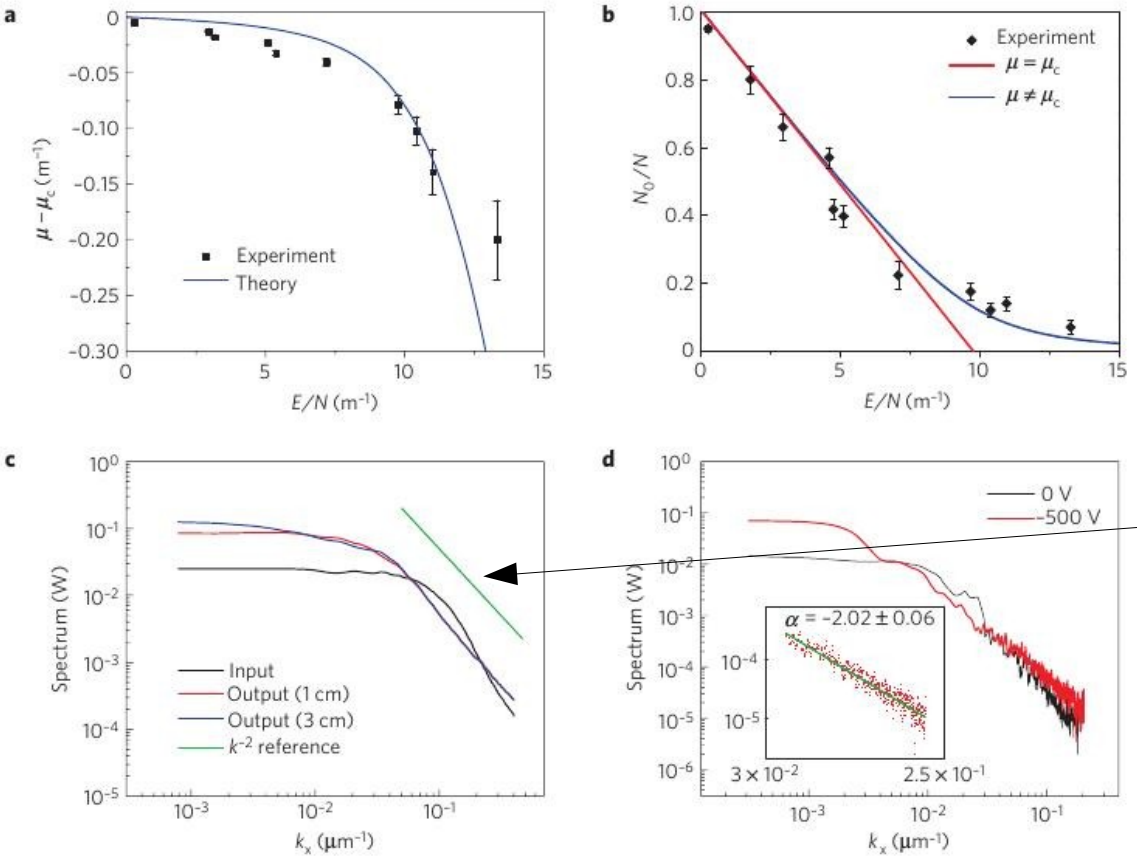
First attempt to measure absence of mechanical effect of superfluid light.

All-optical analog: lateral shift of obstacle beam





Condensation of classical waves

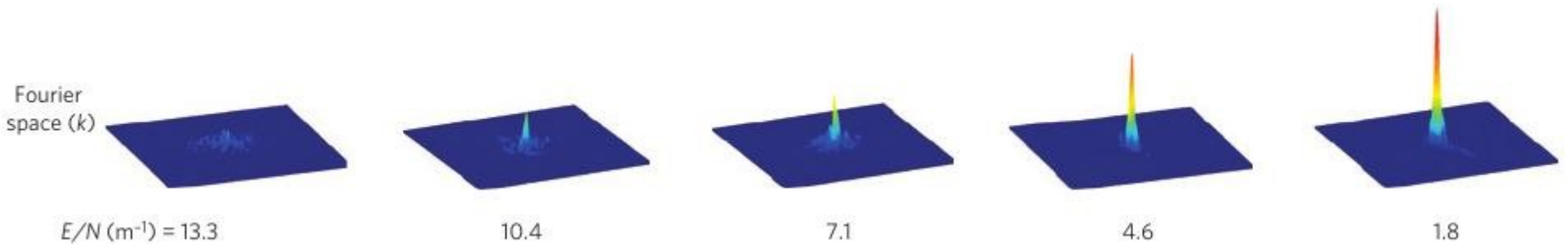


Monochromatic beam
but spatially noisy profile

Slow nonlinearity
→ remains monochromatic
Evolution during propagation
→ classical GPE

Thermalizes to condensate
plus thermal cloud with
Rayleigh-Jeans $1/k^2$ high-k tail

- What about quantum effects?
- How to recover Planckian?



Sun et al., Nature Physics 8, 470 (2012)

How to include quantum fluctuations beyond MF

Requires going beyond monochromatic beam and explicitly including physical time

Gross-Pitaevskii-like eq. for propagation of quasi-monochromatic field with spatially local $\chi^{(3)}$

$$i\frac{\partial E}{\partial z}=-\frac{1}{2\beta_0}\left(\frac{\partial^2 E}{\partial x^2}+\frac{\partial^2 E}{\partial y^2}\right)-\frac{1}{2D_0}\frac{\partial^2 E}{\partial t^2}+V(r)E+g|E|^2E$$

Propagation coordinate z → time

Physical time → extra spatial variable, dispersion D_0 → temporal mass

(similar to Michael's description of light propagation in EIT medium)

Upon quantization → conservative many-body evolution in z: $i\frac{d}{dz}|\psi>=H|\psi>$

with $$H=N\iiint dx\,dy\,dt\left[\frac{1}{2\beta_0}\nabla\hat{E}^\dagger\nabla\hat{E}-\frac{D_0}{2}\frac{\partial\hat{E}^\dagger}{\partial t}\frac{\partial\hat{E}}{\partial t}+V\hat{E}^\dagger\hat{E}+\hat{E}^\dagger\hat{E}^\dagger\hat{E}\hat{E}\right]$$

Same z commutator $$\left[\hat{E}(x.y,t,z),\hat{E}^+(x',y',t',z)\right]=\frac{c\hbar\omega_0 v_0}{\epsilon}\delta(x-x')\,\delta(y-y')\,\delta(t-t')$$

Difficulty for Rydberg-EIT: interactions non-local in x,y, and z → approximated as non-local in t

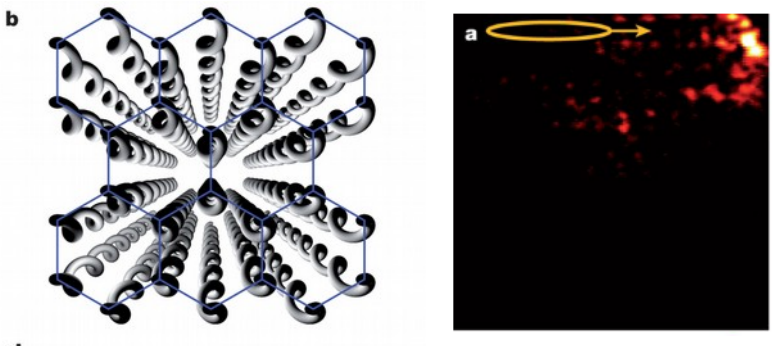
P.-E. Larré, IC, *Propagation of a quantum fluid of light in a cavityless nonlinear optical medium: General theory and response to quantum quenches,* PRA **92**, 043802 (2015)
See also old work by Lai and Haus, PRA 1989

A quite generic *quantum simulator*

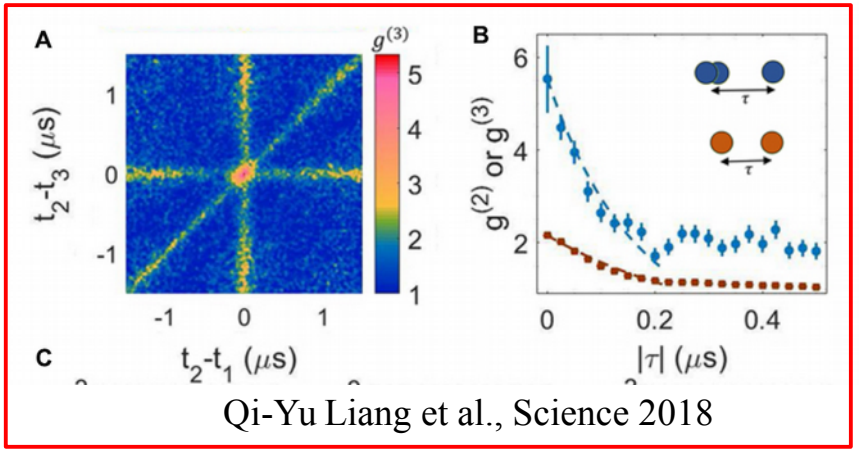
Quantum many-body evolution in z:

$$i\frac{d}{dz}|\psi\rangle = H|\psi\rangle \qquad \text{with:} \qquad H = N\iiint dx\,dy\,dt\left[\frac{1}{2\beta_0}\nabla\hat{E}^\dagger\nabla\hat{E} - \frac{D_0}{2}\frac{\partial\hat{E}^\dagger}{\partial t}\frac{\partial\hat{E}}{\partial t} + V\hat{E}^\dagger\hat{E} + \hat{E}^\dagger\hat{E}^\dagger\hat{E}\hat{E}\right]$$

- Physical time t plays role of extra spatial coordinate
- Same z commutator: $\left[\hat{E}(x,y,t,z),\hat{E}^+(x',y',t',z)\right] = \frac{c\hbar\omega_0 v_0}{\epsilon}\delta(x-x')\,\delta(y-y')\,\delta(t-t')$



Rechtsman et al., Nature 2012



Qi-Yu Liang et al., Science 2018

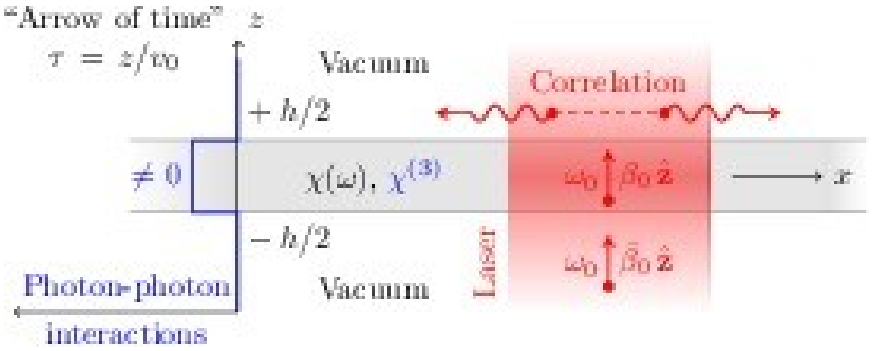
Can realize (or quantum simulate) wide variety of physical systems:

- Arbitrary splitting/recombination of waveguides → quench of tunneling
- Modulation along z → Floquet topological insulators

- In addition to photonic circuit → many-body due to photon-photon interactions
- Pioneering experiments of few-body physics, e.g. two- and three-photon bound states

Dynamical Casimir emission at quantum quench (I)

Monochromatic wave @ normal incidence
into slab of weakly nonlinear medium
→ Weakly interacting Bose gas



Air / nonlinear medium interface
→ sudden jump in interaction constant when moving along z

Mismatch of Bogoliubov ground state in air and in nonlinear medium
→ emission of phonon pairs at opposite k on top of fluid of light
(sort of Dynamical Casimir Effect for phonons)

Propagation along z
→ conservative quantum dynamics

Important question: what is quantum evolution at late times? Thermalization?

P.-E. Larré and IC, PRA **92**, 043802 (2015)

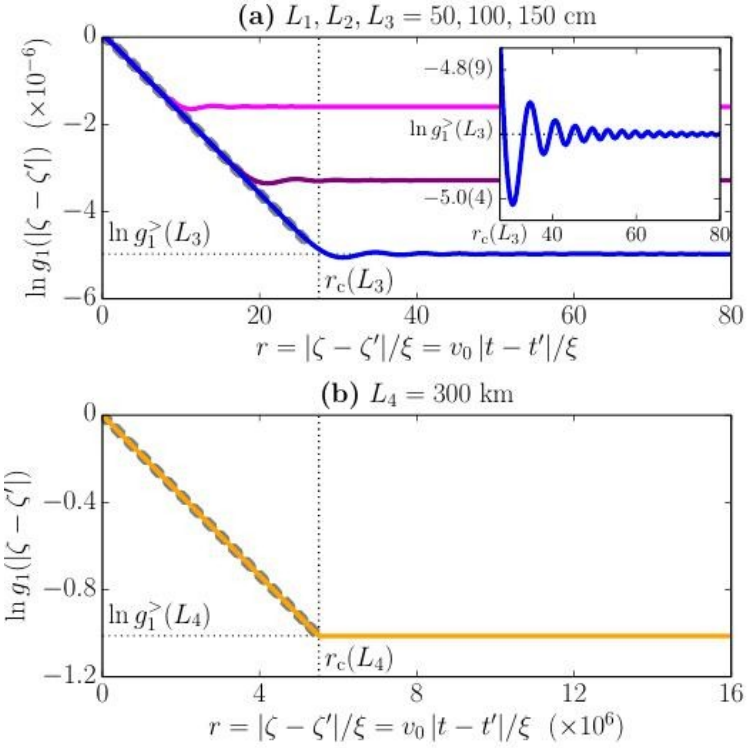
"Pre-thermalized" 1D photon gas

Perfectly coherent light injected into 1D optical fiber:

- quantum quench of interactions ~ $\chi^{(3)}$
- pairs of Bogoliubov excitations generated

Resulting phase decoherence in $g^{(1)}(t\text{-}t')$:

- Exponential decay at short $|t\text{-}t'| < 2z / c_s$
  (c_s = speed of Bogol. sound)
- Plateau at long $|t\text{-}t'| > 2z / c_s$
- Low-k modes eventually tends to thermal $T_{eff} = \mu / 2$
- Hohenberg-Mermin-Wagner theorem prevents long-range order in 1D quasi-condensates at finite T

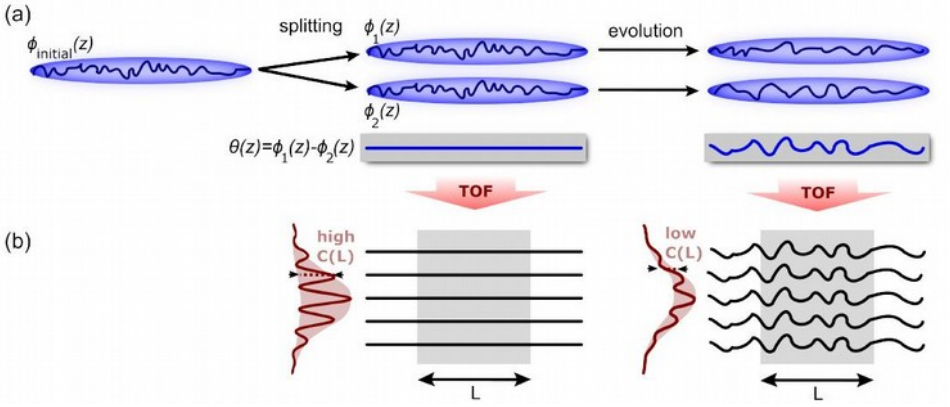


Effect small for typical Si fibers, still potentially harmful on long distances

Decoherence slower if tapering used to "adiabatically" inject light into fiber

Related cold atom expts by J. Schmiedmayer when 1D quasi-BEC suddenly split in two

Nature Physics 9, 640–643 (2013)



P.-E. Larré and IC, *Prethermalization in a quenched one-dimensional quantum fluid of light*, EPJD 2016

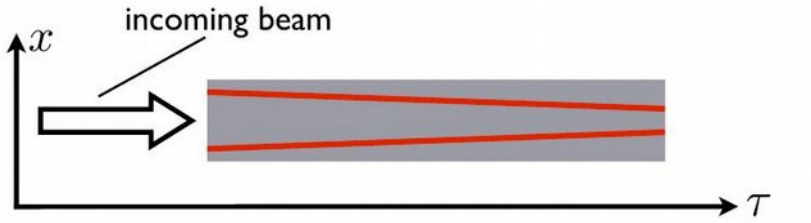
Evaporative cooling of light

Quantum Hamiltonian under space-z / time-t mapping:

$$H = N \iiint dx\ dy\ dt \left[\frac{1}{2\beta_0} \nabla \hat{E}^\dagger \nabla \hat{E} - \frac{D_0}{2} \frac{\partial \hat{E}^\dagger}{\partial t} \frac{\partial \hat{E}}{\partial t} + g \hat{E}^\dagger \hat{E}^\dagger \hat{E} \hat{E} \right]$$

In 3D bulk crystal after long propagation distances:

- equilibration in transverse k and frequency ω leads to Bose-Einstein distribution
- temperature and chemical potential fixed by incident distribution I(k,ω)



Harmonic trap in xy plane + selective absorption of most energetic particles:

- Energy redistributed by collisions; photon gas evaporatively cooled
- Incident incoherent (in both space and time) field eventually gets to BEC state
- NOTE: fast and coherent optical nonlinearity $\chi^{(3)}$ essential !!

Novel source of coherent light

A. Chiocchetta, P.-É. Larré, IC, EPL (2016).
Intriguing (and not yet fully understood) experiment, Krupa et al., Nat. Phot. 2017

Conclusions and perspectives

1-body magnetic and topological effects for photons & atoms in synthetic gauge field:

- Unidirectional and topologically protected edge states of light (2009-)
- Geometrical properties of bulk & anomalous current for atoms & photons (2013-)

New ideas being explored:

- Berry curvature as momentum space magnetic field → toroidal Landau levels, k-space magnetism
- synthetic dimensions → realize high-d models (4d IQHE, …); long vs. short range interactions
- Quantum fluids of light in propagating geometries (*t*-*z* mapping etc.)

Towards many-body physics with light:

- Many platforms for photon blockade: CQED with atoms and solids, circuit-QED,…
- Rydberg blockade easily integrated with synthetic-B in non-planar ring cavities
- Intersubband polaritons in FIR/MIR, large electric dipole moment, large coupling to phonons

- Mott-insulator (recent experiments!), Laughlin states, etc. (expected to come soon!)
- Speculations: anyonic braiding phases. Dream: all-optical topological quantum operations

If you wish to know more...

REVIEWS OF MODERN PHYSICS, VOLUME 85, JANUARY–MARCH 2013

Quantum fluids of light

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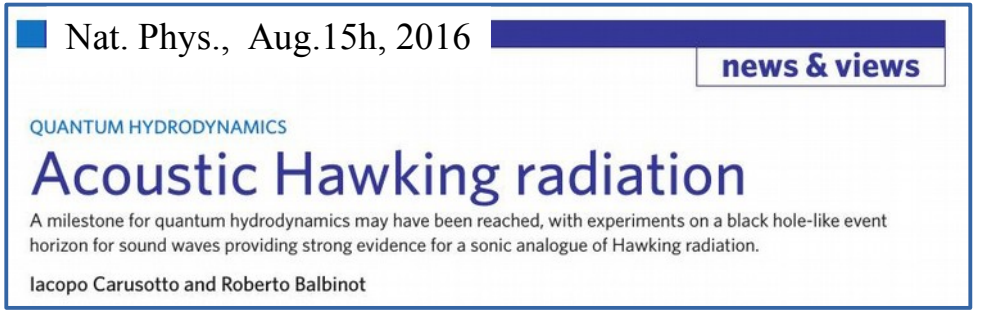
(published 21 February 2013)

I. Carusotto and C. Ciuti, Reviews of Modern Physics **85**, 299 (2013)

BEC
BOSE EINSTEIN CONDENSATION

Come and visit us in Trento!

I. Carusotto, Il Nuovo Saggiatore – SIF magazine (2013)



Topological Photonics

Review article arXiv:1802.04173 by the dream team
Ozawa, Price, Amo, Goldman, Hafezi, Lu, Rechtsman, Schuster, Simon, Zilberberg, IC

PhD positions available starting 10/2018, deadline late Aug, check www.unitn.it website

Call for PostDoc positions in 2019 in PhoQuS project: stay tuned!

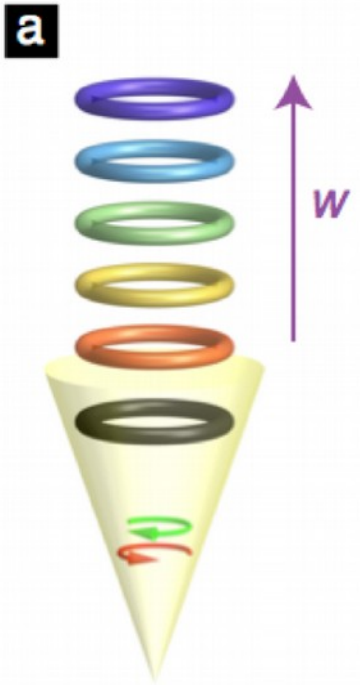
MIR-BOSE

SIQURO

AQψS
ANALOG QUANTUM SIMULATORS FOR MANY-BODY DYNAMICS

Way-out: compactified synthetic dimension

Ring resonators: mode index w spans synthetic dimension,
physically → w = angular momentum around ring



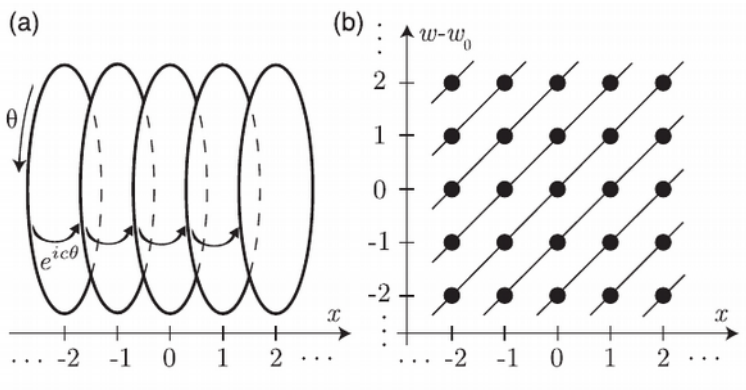
Fourier transform to angle θ:

- w plays now role of momentum.
- Design resonators so to have kinetic energy:

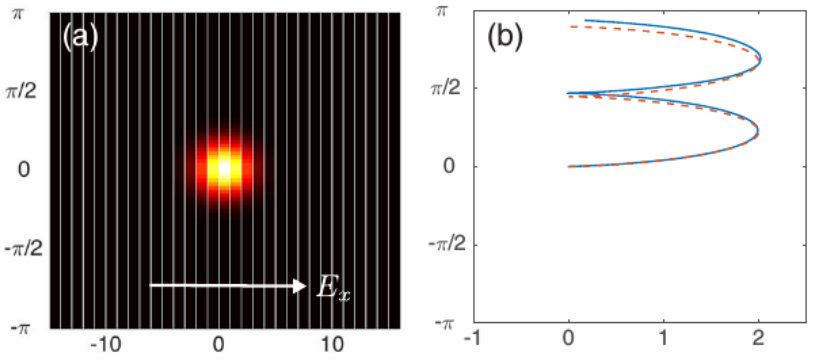
$$\Omega_{x,w} = \Omega_0 + \Omega_{\mathrm{FSR}}(w - w_0 - xc) + D(w - w_0)^2/2$$

- Effective tunneling shows Peierls phase → synthetic B in (x, θ) space
- Cylindrical geometry: (potentially) infinite x, compact θ

As requested: optical nonlinearity (i.e. interactions) local in θ

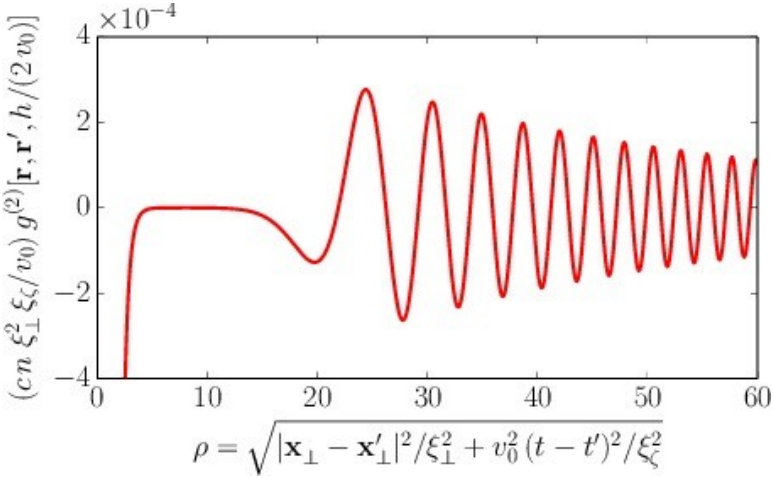


Cyclotron motion



Ozawa-IC, *Synthetic Dimensions with Magnetic Fields and Local Interactions in Photonic Lattices* PRL **118**, 013601 (2017)

Dynamical Casimir emission at quantum quench (II)



Observables:

- Far-field → correlated pairs of photons at opposite angles
- Near-field → peculiar pattern of intensity noise correl.

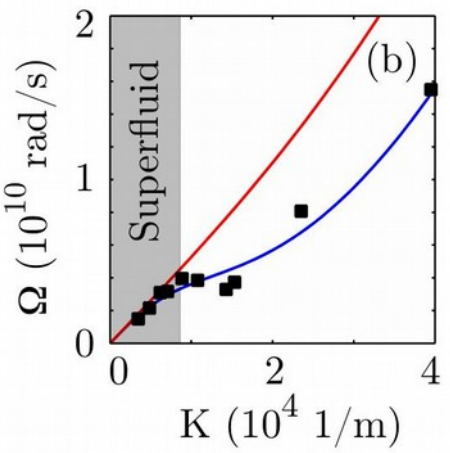
First peak propagates at the speed of sound c_s

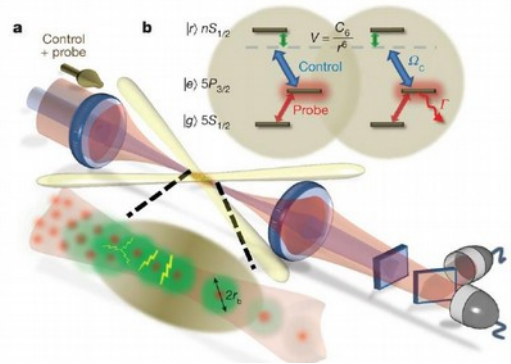
May simulate dynamical Casimir effect & fluctuations in early universe

:

Pump & probe expt for speed of sound c_s :

- c_s^{xy} (Heriot-Watt – Vocke et al. Optica '15)
- c_s^{t} (Trento, in progress)

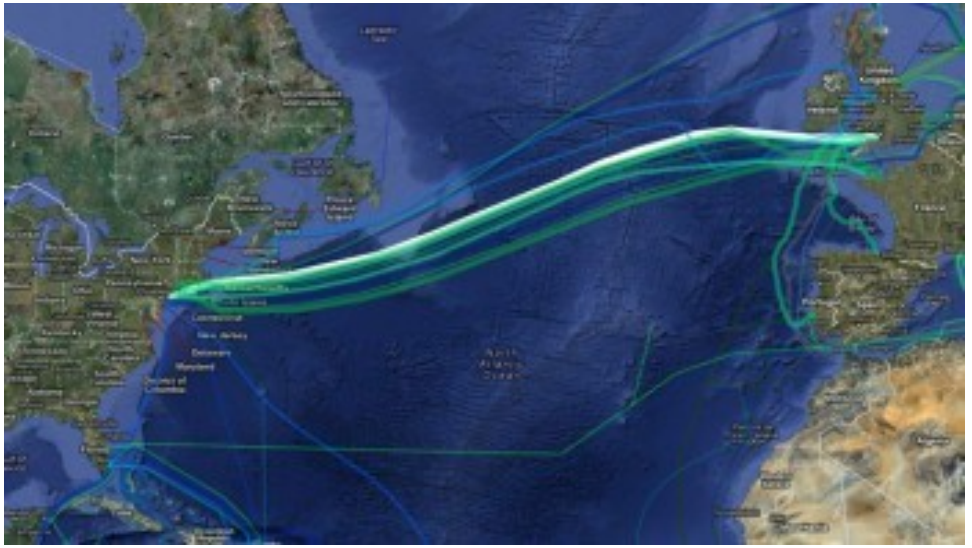




Quantum dynamics most interesting in strongly nonlinear media, e.g. Rydberg polaritons

P.-E. Larré and IC, PRA **92**, 043802 (2015)

A potentially important technological issue...



Long-distance fiber-optic set-ups
→ telecom over distances ~10^4 km

Can optical coherence be preserved?

Several disturbing effects:

- (extrinsic) fluctuations of fiber temperature, length, etc.
- (intrinsic) Fiber material has some (typically weak) $\chi^{(3)}$
  Shot noise on photon number gives fluctuations of $n_{refr} \sim n_0 + \chi^{(3)} I$

  Statistical mechanics suggests that phase fluctuations destroy 1D BEC

  → light at the end of fiber has lost its (temporal) phase coherence

Is this intuitive picture correct? How to tame phase decoherence?